



## Pareto Optimal Design of a Fuzzy Adaptive Robust Fractional-order PID Controller for an Active Suspension System

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### ABSTRACT

This research proposes a robust fuzzy adaptive fractional-order proportional-integral-derivative (PID) controller for an active suspension system of a quarter-car model. For this, the research first designed the PID controller using chassis acceleration and relative displacement. Next, it utilized the chain derivative rule and the gradient descent mechanism to formulate adaptation rules based on integral sliding surfaces. In the next step, the control parameters were regulated by employing a fuzzy system comprising the product inference engine, singleton fuzzifier, and center average defuzzifier. Eventually, the optimum gains of the proposed controller were determined by running a multi-objective material generation algorithm (MOMGA). Simulation results implied the superiority of the proposed controller over other controllers in handling road irregularities.

## 1. Introduction

Over the past years, suspension systems (SSs) have become a subject of interest for much research, primarily owing to their potency to deliver thrilling riding, conserve the chassis against road roughness, lessen the unsuitable operations in steering and braking, and improve the stability of the vehicles (Bagheri et al. 2011). All car manufactures comprehend that comfortable driving is a meaningful option for customers. They further know that car body damage emanates from vibrations caused by road bumps. Accordingly, devising versatile SSs has become a central goal for all car manufacturing companies.

The first SSs developed were passive with a damper and two springs enabling vibrational energy absorption with no need for extra power (Gadhvi et al. 2016). The next generation of such

SSs were semi-active, enhancing driving quality and stability by eliminating pitch and roll movements and aftereffect deviations of the braking. These low-frequency systems were available in soft to hard form and could reduce vibrations in the vehicle chassis and body (Ding et al. 2023). And very recently, active SSs have been proposed to synchronize enjoyable driving, comfortable riding, and improved handling. These systems can save, damp, and generate energy by adjusting their features according to motion conditions (Mahmoodabadi and Nejadkourki 2022).

Over the past decades, research has devised diverse control approaches to handle nonlinear SSs (Chen et al. 2025). Of all the control approaches proposed, proportional-integral-derivative (PID) controllers are sought-after by researches and

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industries owing to their simple structure and application. For instance, a PID controller based on a backpropagation neural network is presented in (Jiang and Cheng 2023) for an air SS, where the research reflects the stiffness of rubber bellows. Elsewhere, PID and fuzzy logic controllers are proposed in (Talib et al. 2023) to experimentally investigate ride comfort SSs by running a firefly algorithm. Similarly, ref. (Kumar and Rana 2023) designs a fuzzy PID controller with an electro-hydraulic actuator for nonlinear active SSs, where the cost function is analyzed by fifty runs. A Pareto optimality-based PID controller has been employed in (Gampa et al. 2023) for a vehicle active SS by running the grasshopper optimization algorithm, with the aim of minimizing sprung mass acceleration, tire deflection, and sprung mass suspension deflection. Ref. (Li et al. 2022) devises an improved fuzzy neural network PID controller for an active SS, where body acceleration is pondered as the main optimization target.

Merging fuzzy logic-based systems with controllers can synergistically augment their potency by utilizing human knowledge in the real world. The two key facets reflected by these controllers are insensitivity to environmental factors and robustness. Recent research advocates the efficacy of running fuzzy systems for tuning parameters of control approaches. Some examples are optimal fuzzy PID control (Melese et al. 2025), nonlinear fuzzy PID control (Mohindru 2024), adaptive fuzzy (AF) sliding mode (SM) control (Zheng et al. 2024), self-AF PID control (Abut and Soyguder 2022), fuzzy embedded PID control (Venkataramanan et al. 2024), and AF backstepping control (Liu et al. 2023).

When devising controllers, fractional-order (FO) derivatives and/or integrals can provide the systems with a versatile performance. Oustloup (1988) first suggested applying FO calculus for controllers. Next, Podlubny (1994) introduced a FOPID approach as an illustrious FO controller (Dadras and Momeni 2012). Research has implied the efficiency of merging FO concepts with control schemes. In (Nosheen et al. 2023), FO calculus has been utilized to develop a sensorless speed control aided with an extended Kalman filter for a closed loop induction motor. Ref. (Naderipour et al. 2023) has devised an optimal load-frequency self-tuning FO fuzzy controller for an islanded microgrid to attain robust performance and satisfying flexible structure. An adaptive FOSM disturbance observer (DO)-based robust theoretical frequency controller has been utilized in (Guha et al. 2023) for a hybrid wind–diesel power system to minimize chattering in the control effort and attain promoted robustness

against external disturbances. Ref. (Liu et al. 2023) has employed a FO controller based on digital-twin-based real-time optimization for industrial robots modeled in a digital environment to realize all-around 3D visual monitoring and strong interactive. Elsewhere, ref. (Ansarian and Mahmoodabadi 2023) reports a Fuzzy Adaptive (FA) robust FOPID controller optimized by a MO approach for a nonlinear unmanned flying system, where the fuzzy systems employ the product inference engine, center average defuzzifier, singleton fuzzifier, and triangular-trapezoidal membership functions.

In real applications of controller design, balancing output errors and control efforts is paramount. Metaheuristic (MH) algorithms mostly utilized to optimize MO problems have recently been sought-after by researchers (Peng et al. 2025). Research implies the potency of these algorithms in solving MO control problems. Ref. (Wang et al. 2023) reports a MO approach for a hydrogen-fueled rotary engine based on GA and machine learning (ML). Ref. (Tamashiro et al. 2023) employs an optimal components capacity based MO scheme and an optimal scheduling-based MPC optimization algorithm for smart apartment buildings. In (Zhou et al. 2023), an adaptive adjustment inertia weight particle swarm optimization (PSO) algorithm has been run used for the optimal design of a cable-driven parallel robot. Ref. (Abedzadeh Maafi et al. 2021) uses an MOGA for the optimal design of a FA hierarchical SM controller of an XZ inverted pendulum system. Ref. (Kapnopoulos et al. 2022) uses a cooperative PSO strategy containing position and attitude control parameters for gain tuning of an MPC-based quadrotor trajectory tracking scheme.

To enhance the breadth of recent advancements, this study also considers the following contributions:

- Multi-criteria suspension optimization (Gheibollahi and Masih-Tehrani 2023)
- Fuzzy Sliding mode control in vehicle dynamics (Najafi et al. 2023)
- Genetic algorithms for vehicle design (Gheibollahi et al. 2024)
- Stability under varying road conditions (Damavandi et al. 2022)
- Semi-active suspension energy efficiency (Nazemian and Masih-Tehrani 2020)
- ISO-based road roughness modeling (Nazemian and Masih-Tehrani 2020)
- Vehicle dynamics under uncertainty (Nazemi et al. 2022)
- Recent  $H_{\infty}$ -based suspension control schemes (Damavandi et al. 2025)

•Robustness in suspension design (Najafi and Masih-Tehrani 2022).

Contrary to (Bagheri et al. 2011, Ding et al. 2023) applying passive and semi-active SSs, the present research proposes a hybrid controller by merging FOPID compensators, sliding surfaces, and fuzzy systems. This controller aims to stabilize a nonlinear active SS. Hence, this research devises a PID controller improved by FO concepts to handle the nonlinear dynamics in the concerned active SS. The sliding surfaces are calculated via the system states to adapt the gains of the FOPID controller according to the system conditions. This research employs a fuzzy system based on the center average defuzzifier, singleton fuzzifier, and product inference engine to regulate the sliding surface parameters. Contrary to (Talib et al. 2023, Kumar and Rana 2023) that employ single objective optimization, this research (after designing the structure of the proposed control system) uses the MO material generation algorithm (MOMGA) to determine the optimal values of the adaptation coefficients, fuzzy system parameters, and fractional orders. The aim is to coincidentally minimize body acceleration and relative displacement. Eventually, simulation results are depicted to illustrate the potency of the proposed method in handling the concerned active SS compared to other controllers.

The remaining sections are organized as follows. Section 2 discusses the dynamical formulations of the active SS. Section 3 explain adaptive FOPID controllers and fuzzy systems. Section 4, runs the MOMGA to define the appropriate values of the design coefficients. Section 5 presents the simulation results to show the superiority of the proposed method over recently introduced methods. Ultimately, Section 5 concludes the results and offers prospects.

## 2. Dynamical Equations

The dynamical equations of the active SS shown in Figure 1 can be written as follows (Mahmoodabadi and Nejadkourki 2022).

$$m_r \ddot{z}_r(t) + c_s [\dot{z}_r(t) - \dot{z}_s] + k_s [z_r(t) - z_s(t)] + k_r [z_r(t) - z_w(t)] = -\tau(t) \quad (1)$$

$$m_s \ddot{z}_s(t) + c_s [\dot{z}_s(t) - \dot{z}_r] + k_s [z_s(t) - z_r(t)] = \tau(t) \quad (2)$$

where  $\tau(t)$  denotes the control effort.  $z_r(t)$  and  $z_s(t)$  are, respectively, the vertical displacement of the tire and sprung mass,  $\dot{z}_r(t)$  and  $\dot{z}_s(t)$  denote their vertical velocities, and  $\ddot{z}_r(t)$  and  $\ddot{z}_s(t)$  represent their vertical acceleration. In addition,

$m_r$ ,  $m_s$ ,  $c_s$ ,  $k_s$ , and  $k_r$  define the tire mass, sprung mass, damping coefficient, spring stiffness coefficient, and tire stiffness coefficient, correspondingly. Ultimately,  $z_w(t)$  indicates the vertical displacement caused by irregularities that can be formulated as follows.

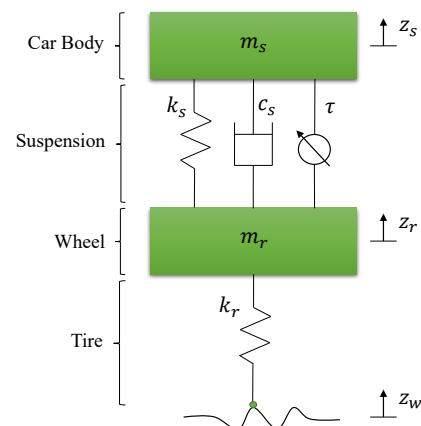
$$z_w(t) = \begin{cases} 0.05[\sin(2\pi t)] & \text{if } 0.5 < t < 2.5 \\ 0 & \text{else} \end{cases} \quad (3)$$

In this study, a deterministic road input defined by Equation (3) is used to evaluate the baseline performance of the proposed controller. Although more realistic stochastic road profiles can be generated based on ISO 8608 standards, they were not considered in this initial model to isolate the controller's performance under controlled conditions. Incorporating ISO-compliant random road excitations is proposed as a direction for future work.

It is also worth noting that the employed quarter-car model only captures vertical dynamics. Hence, the effects of lateral load transfer and roll dynamics during cornering maneuvers are not reflected in the current formulation. Extension to a half-car or full-vehicle model would be necessary to account for such phenomena in future studies.

**Table 1. Values of the constant coefficients for the studied vehicle SS according to Ref. (Mahmoodabadi and Nejadkourki 2022) .**

coefficient	value	unit
$m_r$	36	kg
$m_s$	240	kg
$c_s$	1978	kNs/m
$k_r$	160000	kN/m
$k_s$	10529	kN/m



**Figure 1: A schematic representation of the studied quarter-car active SS.**

Table 1 tabulates the numerical values of the constant parameters in Equations (1) and (2) and

compares these results with those reported in Ref. (Mahmoodabadi and Nejadkourki 2022).

### 3. Controller Design

#### 3.1. Fractional-order PID Controller

The PID, as a widely known controller, is extensively employed owing to its efficacy and simplicity (Aessa et al. 2023). Recently, response speed and robustness have been reported in the FO version of this compensator (El-Rifaie et al. 2025, Sahin et al. 2024). Some definitions of the FO operators are described in the following (Ansarian and Mahmoodabadi 2023).

**Definition 1.** Caputo fractional of  $\lambda^{th}$  order derivative and integral for function  $k(t)$  are given by the following equations.

$${}_t^C D_t^\lambda k(t) = \frac{1}{\Gamma(\lambda-\gamma)} \int_{t_0}^t \frac{k^{(\gamma)}(\tau)}{(t-\tau)^{\lambda-\gamma+1}} d\tau \quad (4)$$

$${}_t^C I_t^\lambda k(t) = \frac{1}{\Gamma(\lambda)} \int_{t_0}^t \frac{k(\tau)}{(t-\tau)^{1-\lambda}} d\tau \quad (5)$$

where  $\gamma - 1 < \lambda \leq \gamma$ , and  $\gamma$  represents the smallest integer number greater than  $\lambda$ . Further,  $t$  and  $t_0$  are the final and initial time values, respectively reflecting the upper and lower bounds of integration.  $\Gamma(\cdot)$  denotes the Gamma function written by the following equation.

$$\Gamma(\omega) = \int_0^\infty e^{-t} t^{\omega-1} dt \quad (6)$$

Therefore, a common version of the FOPID controller for a system with two errors  $e_1$  and  $e_2$  can be revealed as follows.

$$\tau(t) = \tilde{\eta}_{p1} e_1(t) + \tilde{\eta}_{i1} I^\nu e_1(t) + \tilde{\eta}_{d1} D^\rho e_1(t) + \tilde{\eta}_{p2} e_2(t) + \tilde{\eta}_{i2} I^\nu e_2(t) + \tilde{\eta}_{d2} D^\rho e_2(t) \quad (7)$$

where  $\tilde{\eta}_{p1}$ ,  $\tilde{\eta}_{i1}$ ,  $\tilde{\eta}_{d1}$  and  $\tilde{\eta}_{p2}$ ,  $\tilde{\eta}_{i2}$ ,  $\tilde{\eta}_{d2}$  are proportional, integral, and derivative coefficients related to the first and second errors, respectively. Correspondingly,  $D^\rho$  and  $I^\nu$  are Caputo fractional derivative and integral of orders  $\rho$  and  $\nu$ .

#### 3.2. Adaptive Robust Fractional-order PID controller

When designing a controller, a key aspect is to attain appropriate values for the control gains. For this, adaptive methods are supposedly highly potent to timely set these gains. Furthermore, robust SM techniques can be utilized to modify time-dependent relations of the adaptive approach. Thus, integral sliding surfaces related to a system with two errors can be calculated from the equation below (Slotine and Li 1991).

$$s_j = e_j + \tilde{c}_j \int e_j dt, \quad j = 1, 2 \quad (8)$$

where  $e_j$   $j = 1, 2$  denotes the system errors based on the system states, and  $\tilde{c}_j$   $j = 1, 2$  represent sliding surface parameters that can be obtained from the equation below.

$$\tilde{c}_j = \hat{c}_j + c_j, \quad j = 1, 2 \quad (9)$$

where,  $c_j$   $j = 1, 2$  are positive constant parameters, while  $\tilde{c}_j$   $j = 1, 2$  signify fuzzy parameters. The time-dependent adaptive part of the proportional, integral, and derivative coefficients can be calculated utilizing the gradient descent technique as follows (Astrom and Wittenmark 2008).

$$\dot{\tilde{\eta}}_{pj} = -\tilde{\alpha}_{pj} s_j e_j, \quad j = 1, 2 \quad (10)$$

$$\dot{\tilde{\eta}}_{ij} = -\tilde{\alpha}_{ij} s_j \int e_j dt, \quad j = 1, 2 \quad (11)$$

$$\dot{\tilde{\eta}}_{dj} = -\tilde{\alpha}_{dj} s_j \frac{de_j}{dt}, \quad j = 1, 2 \quad (12)$$

where,  $\tilde{\alpha}_{pj}$ ,  $\tilde{\alpha}_{ij}$  and  $\tilde{\alpha}_{dj}$  can be regulated by fuzzy parameters  $\hat{\alpha}_{pj}$ ,  $\hat{\alpha}_{ij}$  and  $\hat{\alpha}_{dj}$ , respectively.

$$\tilde{\alpha}_{xj} = \hat{\alpha}_{xj} + \alpha_{xj}, \quad j = 1, 2, x = p, i, d \quad (13)$$

where,  $\alpha_{xj}$   $j = 1, 2$  and  $x = p, i, d$  denote positive constant parameters. Ultimately, the coefficients of the FOPID controller introduced in Equation (7) can be clarified as follows.

$$\tilde{\eta}_{xj} = \eta_{xj} + \bar{\eta}_{xj} \quad j = 1, 2 \text{ and } x = p, i, d \quad (14)$$

where,  $\eta_{xj}$   $j = 1, 2$  and  $x = p, i, d$  indicate positive constant parameters..

#### 3.3. Fuzzy adaptive robust fractional-order PID controller

In the previous subsection, a series of parameters arose when adapting the FOPID control gains. Hence, one potent solution to regulate their values based on the system conditions is to apply fuzzy logic-based systems. As with Equations (10-12), the sliding surfaces influence the overall behavior of the control approach. Therefore, regulating their parameters can affect the system's performance. This research employs fuzzy systems to regulate positive control parameters  $\hat{c}_j$  ( $j = 1, 2$ ) and  $\hat{\alpha}_{xj}$  ( $j = 1, 2$  and  $x = p, i, d$ ). The studied fuzzy systems employ the product inference engine, singleton fuzzifier, and center average defuzzifier, which are formulated as follows (Driankov et al. 2013).

$$\text{fuzzy parameter} = \frac{\sum_{k=1}^N \bar{f}^k(\mu^k(e_k))}{\sum_{k=1}^N (\mu^k(e_k))} \quad (15)$$

where  $N_1$  and  $N_2$  are the numbers of the fuzzy rules and equal to 3, and  $\bar{f}^{k_1 k_2}$  represents the center of

the output membership functions. Ultimately,  $\mu^k$  denotes the input membership function that is pondered as a triangular shape (Figure 2). Furthermore, the fuzzy rules related to the control parameters are given in Table 2.

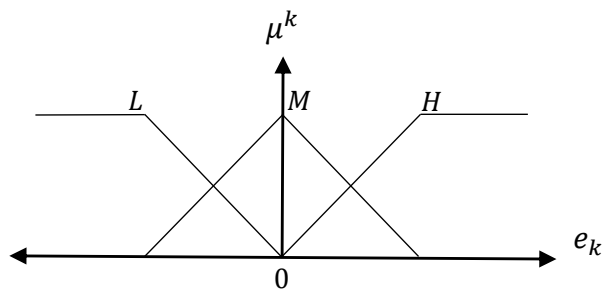


Figure 2. Triangular membership functions utilized for the inputs of the fuzzy systems.

Table 2. Fuzzy rules for fuzzy parameters  $\bar{f}^k$ .

	<i>L</i>	<i>M</i>	<i>H</i>
$\hat{c}_1$	96.04	49.98	96.04
$\hat{c}_2$	18876	229	13.44
$\hat{\alpha}_{p1}$	1489	6157	2401
$\hat{\alpha}_{i1}$	67110	978	984
$\hat{\alpha}_{d1}$	5497	1520	3326
$\hat{\alpha}_{p2}$	24990	1092	17767
$\hat{\alpha}_{i2}$	834.5	20.99	201.6
$\hat{\alpha}_{d2}$	0.099	0.014	0.093

#### 4. Optimization by the MOMGA

Over the last decades, MH algorithms have become popular due to their simplicity, flexibility, derivation-free mechanism, and local optima avoidance (Sadeghian et al. 2025). These features qualify MH optimization approaches as potent solves of the real-world issues, particularly controller design problems (Zhuang et al. 2024). David Schaffer proposed the theory of MO optimization for problems with diverse and potentially conflicting objectives (Schaffer and Grefenstette 1985). Hence, a MO optimization technique based on the material generation algorithm (MGA) can find the optimum values of control gains.

##### 4.1. MGA

Inspired by the configuration of chemical reactions and compounds in producing new materials, the MGA offers highly competitive and even eminent results compared to other MH algorithms (Talatahari et al. 2021). Alike natural evolution algorithms that build a predefined population of solution candidates, MGA determines a number of materials ( $MT$ ), comprised

of multiple periodic table elements ( $Es$ ). A general mathematical modeling of this algorithm is as follows (Talatahari et al. 2021).

$$MT = \begin{bmatrix} E_1^1 & E_1^2 & \cdots & E_1^i & \cdots & E_1^b \\ E_2^1 & E_2^2 & \cdots & E_2^j & \cdots & E_2^b \\ \vdots & \vdots & & \vdots & & \vdots \\ E_j^1 & E_j^2 & \cdots & E_j^i & \cdots & E_j^b \\ \vdots & \vdots & & \vdots & & \vdots \\ E_m^1 & E_m^2 & \cdots & E_m^i & \cdots & E_m^b \end{bmatrix} \quad (16)$$

$$E_j^i(0) = E_{j,min}^i + urand(0,1) (E_{j,max}^i - E_{j,min}^i) \quad (17)$$

$$MT_{new} = [E_{new}^1 \ E_{new}^2 \ \cdots \ E_{new}^s \ \cdots \ E_{new}^b] \quad (18)$$

$$E_{new}^s = E_{r_1}^{r_2} \pm pc \quad (19)$$

$$f(E_{new}^s | \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (20)$$

where  $b$  is the dimension of the problem,  $m$  is the total number of solution candidates,  $E_j^i(0)$  denotes the initial amount of the  $i$ th periodic table element in the  $j$ th material,  $urand(0,1)$  signifies a random number uniformly distributed between  $[0,1]$ ,  $E_{j,min}^i$  and  $E_{j,max}^i$  respectively indicate the minimum and maximum acceptable amounts of the  $i$ th decision variable in the  $j$ th solution candidate,  $r_1$  and  $r_2$  correspondingly denote random integers uniformly spread in  $[1, m]$  and  $[1, b]$ ,  $PT_{r_1}^{r_2}$  (randomly chosen from the  $MT$ ,  $MT_{new}$ ) represents the new generated material,  $PT_{new}^s$  is the new periodic table element, and  $pc$  is the probabilistic component regulating the process of gaining, losing, or even sharing electrons.

##### 4.2. MOMGA

MGA was originally introduced to address single-objective optimization problems and cannot be directly used to solve MO issues. MGA, in turn, requires three mechanisms for solving MO optimization problems. The first mechanism is the archive, a storage space for storing or restoring the obtained Pareto optimal solutions. The second effective approach to enhance non-dominated solutions in the archive is the grid mechanism. When the archive is full, the grid mechanism rearranges the object space's segmentation and finds the most populated zone to prune it. The leader mechanism is the last method introduced to the MGA, which is used to compare solutions in the MO search space. Such a selection is based on the roulette-wheel technique with the following probability:

$$T_i = \frac{p}{n_i} \quad (21)$$

where  $p$  is a constant number higher than 1, and  $n_i$  denotes the variety of obtained Pareto optimal solutions in the  $i$ th section (Nouhi et al. 2022). The MOMGA flowchart is illustrated in Figure 3.

Ultimately, the block diagram of the optimal FA robust FOPID controller is demonstrated in Figure 4.

### 5. Simulation results and discussion

The MOMGA was utilized to find the best values for control parameters, including the positive constant parameters in Equations (9), (13), and (14), the initial conditions for the adaptive gains in Equations (10) through (12), the FO operators in Equation (7), and the centers of the output membership functions in Equation (15). The controller optimization process was formulated as a multi-objective problem with two primary objectives:

(1) Minimization of the integral of absolute relative displacement

$$f_1 = \int |z_s - z_r| dt \quad (22)$$

which reflects road holding performance, and

(2) Minimization of the integral of absolute chassis acceleration

$$f_2 = \int |\ddot{z}_s| dt \quad (23)$$

which is directly related to ride comfort in accordance with ISO 2631 guidelines. These two criteria represent the widely accepted trade-off in suspension design, ensuring both stability and passenger comfort.

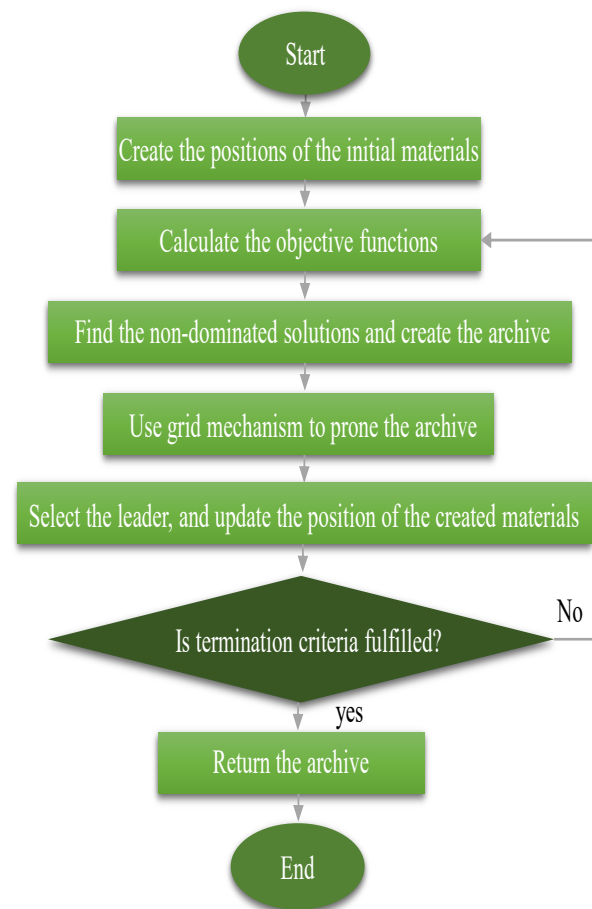


Figure 3. Flowchart of the MOMGA.

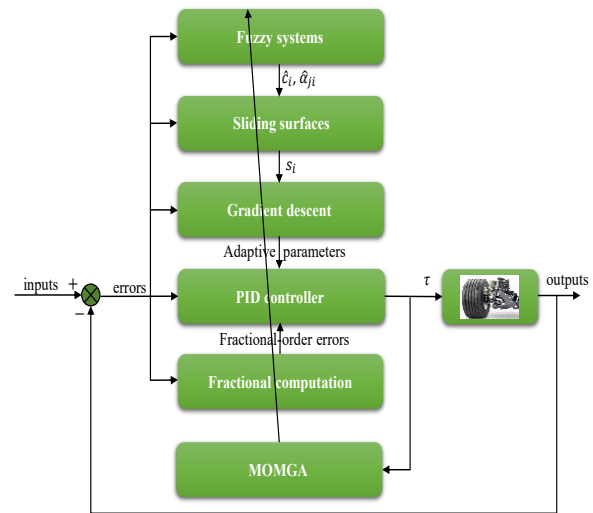


Figure 4. Block diagram of the introduced optimal fuzzy adaptive robust fractional-order PID controller.

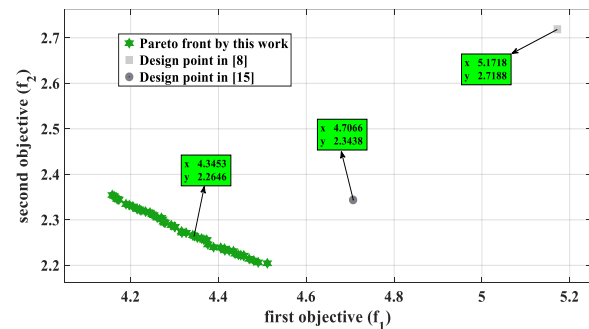


Figure 5. Pareto front as well as the design points obtained by this strategy and suggested in (Bagheri et al. 2011, Mahmoodabadi and Nejadkourki 2022).

Figure 5 depicts the Pareto front achieved through the optimization process and the chosen point, and compares them with the optimum points in (Bagheri et al. 2011, Mahmoodabadi and Nejadkourki 2022). Ref. (Bagheri et al. 2011) has used the MOMGA for Pareto optimization of the two-degree-of-freedom passive linear SS regarding these two conflicting functions. Likewise, Ref. (Mahmoodabadi and Nejadkourki 2022) has designed an FA robust PID controller optimized by a single-objective PSO algorithm for the quarter-car model with an active SS.

To fairly compare these results with those in (Rodriguez-Guevara et al. 2023, Bagheri et al. 2011), the values of the constant parameters for the concerned vehicle SS are reflected based on the data in Table 1 for all methods. Compared to the solutions related to the designed points proposed in (Bagheri et al. 2011, Mahmoodabadi and Nejadkourki 2022), the chosen point from the



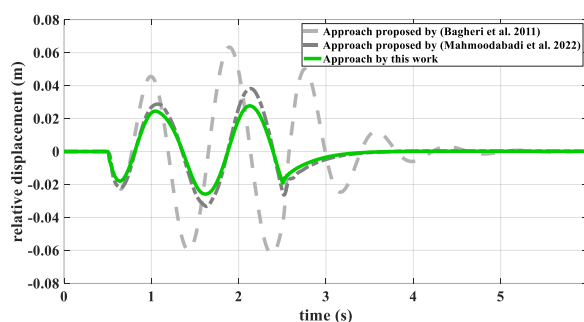
Pareto front has been improved by 8% and 19% (in the relative displacement) and by 3% and 20% (in the chassis acceleration), respectively. Such improvements enhance both road holding and ride comfort, and the resulting chassis acceleration levels fall within ISO 2631 comfort thresholds for light vehicles. Table 3 presents the optimum values of the defined variables, including the initial values of the adaptive coefficients, the FO parameters, and the constant parameters of the gradient descent formulations. Figures 6 and 7 depict the time responses of relative displacement and the chassis acceleration, respectively. As shown, the proposed approach has less relative displacement and has been improved by 27% and 104% (in the overshoot values) and by 5% and 45% (in the settling time values), compared to the methods designed in (Bagheri et al. 2011, Mahmoodabadi and Nejadkourki 2022). Figures 8 to 11 show displacements and velocities of the tire and chassis for the different methods. As demonstrated, the proposed optimal FA robust FOPID control system takes less overshoot values and more efficiently suppresses vehicle vibrations, compared to the control approaches in (Bagheri et al. 2011, Mahmoodabadi and Nejadkourki 2022). The results indicate that the proposed FA robust FOPID controller effectively suppresses oscillations in both sprung and unsprung masses. Although the optimization emphasizes ride comfort, the achieved improvements do not significantly compromise the response time or overshoot. This balance reflects the inherent trade-off in suspension design and highlights the controller's effectiveness in tuning fractional gains adaptively. Figure 12 shows the behaviors of the sliding surfaces without chattering and acceptable ranges. Ultimately, Figures 13 and 14 respectively show variations in the fuzzy and adaptive gains related to the proportional, integral, and derivative terms over time. These diagrams imply the convergence of all parameters to constant values at a reasonable period. It should be noted that the quarter-car model employed in this study does not capture lateral dynamics such as roll or yaw, which are relevant during cornering maneuvers. Therefore, the current analysis is limited to vertical excitations. Future research should extend the model to half-car or full-car configurations to enable comprehensive validation under combined vertical and lateral scenarios.

To evaluate the robustness of the controller against parameter variations, a sensitivity analysis was conducted. The sprung mass was varied within  $\pm 10\%$  of its nominal value as reported in Table 1. This range reflects common tolerances observed in

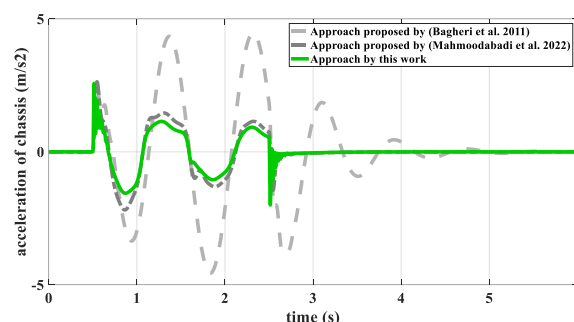
practical applications and component variability. The system's performance under this uncertainty illustrated in Figure 15 was used to assess the controller's robustness in terms of stability and comfort.

**Table 3. Optimum values of the control parameters according to the suggested design point.**

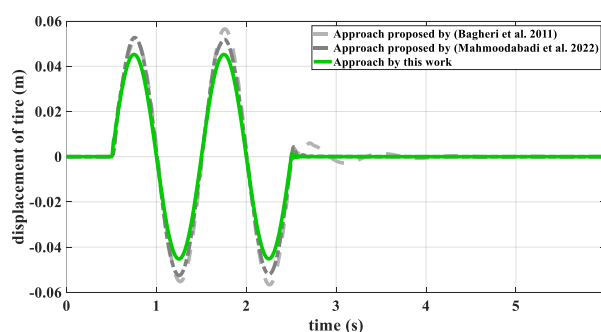
Variable	Value	Variable	Value
$c_1$	67626	$\eta_{i2}$	19.95
$c_2$	416.16	$\eta_{d2}$	0.480
$\alpha_{p1}$	388.96	$\bar{\eta}_{p1}(0)$	3128.98
$\alpha_{i1}$	4.99	$\bar{\eta}_{i1}(0)$	3043.50
$\alpha_{d1}$	125.94	$\bar{\eta}_{d1}(0)$	3164.80
$\alpha_{p2}$	0.960	$\bar{\eta}_{p2}(0)$	48.99
$\alpha_{i2}$	2.895	$\bar{\eta}_{i2}(0)$	503.54
$\alpha_{d2}$	9.996	$\bar{\eta}_{d2}(0)$	0.007
$\eta_{p1}$	416.16	$\nu$	0.916
$\eta_{i1}$	496.57	$\rho$	1.163
$\eta_{d1}$	499.80	$\eta_{p2}$	145.65



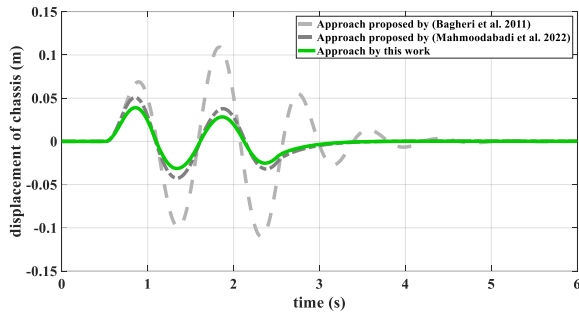
**Figure 6. Time changes of the relative displacement.**



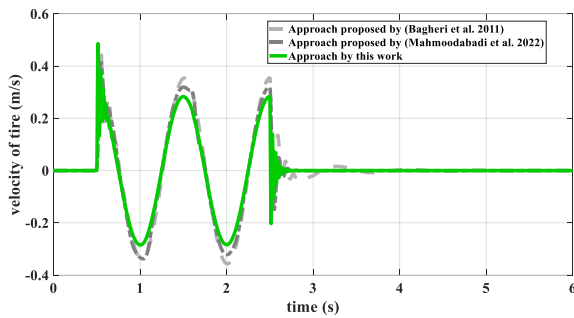
**Figure 7. Time changes of the acceleration of chassis.**



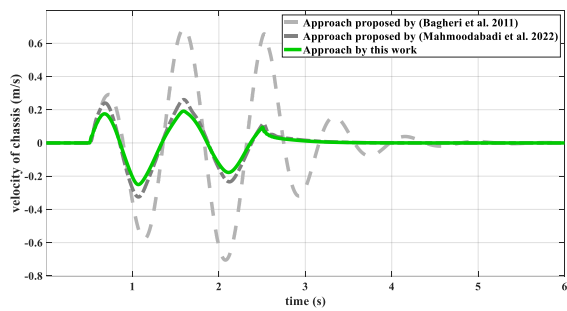
**Figure 8. Time changes of the displacement of tire.**



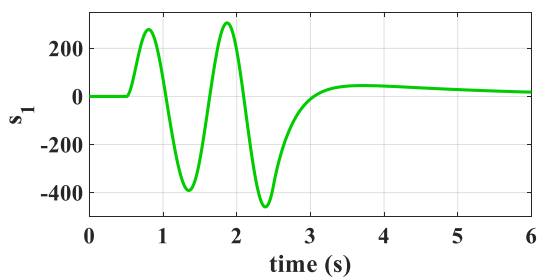
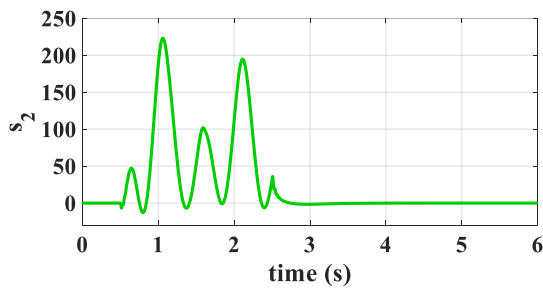
**Figure 9. Time changes of the displacement of chassis.**



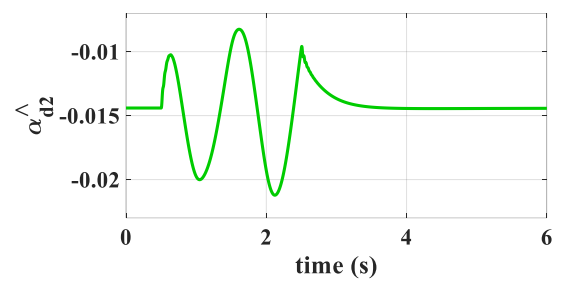
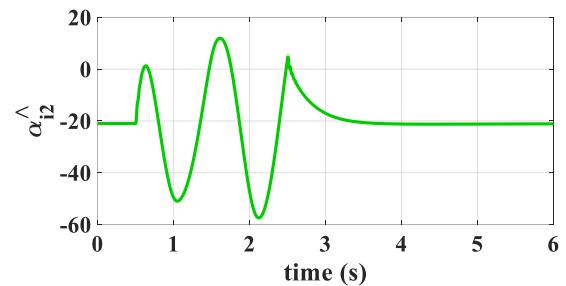
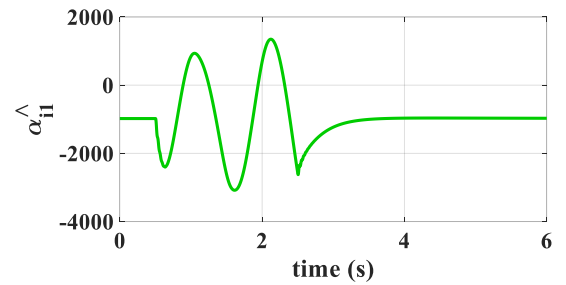
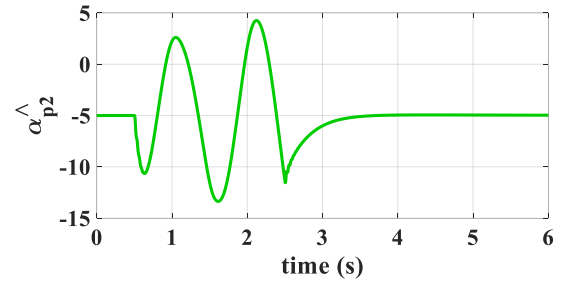
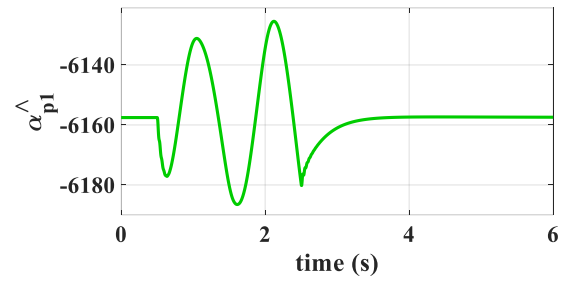
**Figure 10. Time changes of the velocity of tire.**



**Figure 11. Time changes of the velocity of chassis.**



**Figure 12. Time variations of the sliding surfaces for the proposed method of this work.**



**Figure 13. Time variation of the fuzzy gains for the proposed method of this work.**



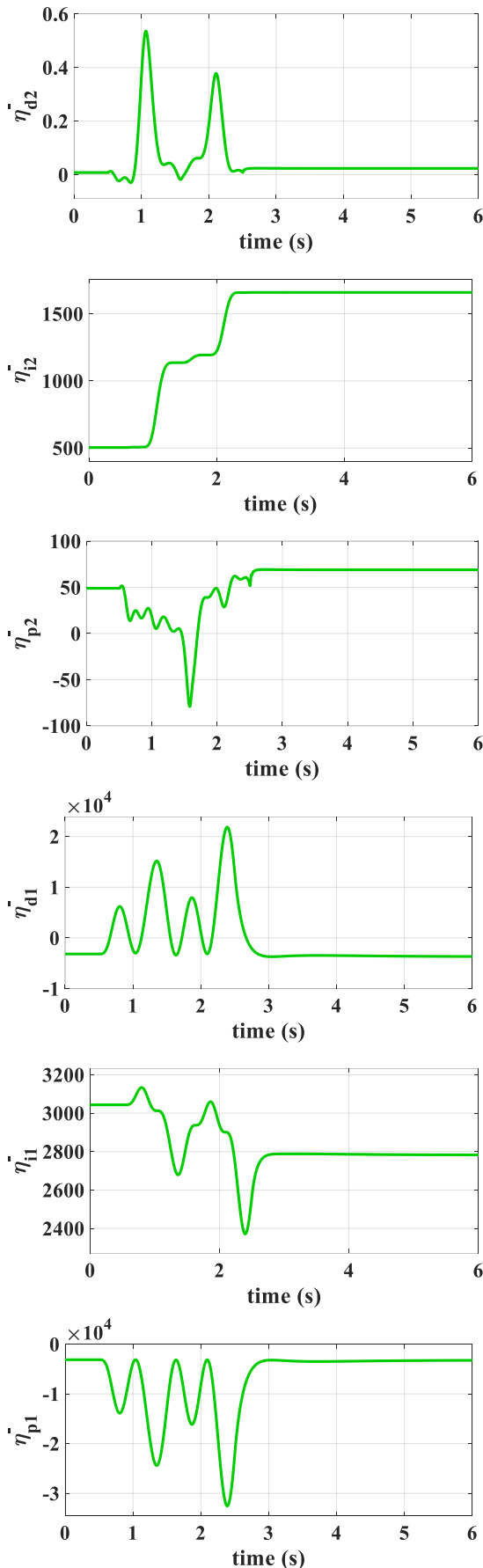


Figure 14. Time responses of the adaptive gains for the proposed method of this work.

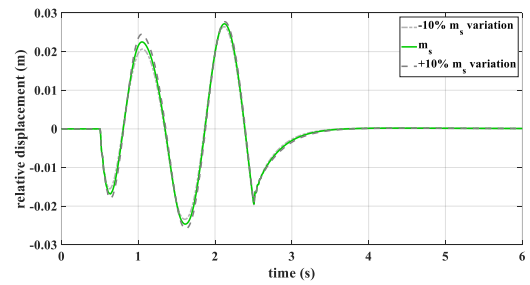


Figure 15. Time changes of the relative displacement over  $\pm 10\%$   $m_s$  variation.

## 6. Conclusions and Future Works

This research proposed a hybrid controller constituting FOPID compensators, sliding surfaces, and fuzzy systems for an active SS. Firstly, a PID controller improved by the FO concepts was proposed to handle the dynamics of the studied nonlinear active SS. Next, the sliding surfaces were formulated based on the relative displacement and chassis acceleration to adaptively compute the gains of the FOPID controller. Then, Fuzzy systems based on the center average defuzzifier, singleton fuzzifier, and product inference engine were used to regulate the parameters of the sliding surfaces and gradient descent equation. After the control system was structurally designed, the MOMGA was run to determine the optimal values of the constant gains, aiming to simultaneously minimize body acceleration and relative displacement. Simulation results revealed the potency of the propose idea in handling vibrations in the active SS, compared to ot her controllers. Regarding these, future research needs to (1) apply other novel MOMGAs to obtain more accurate non-dominate solutions, (2) utilize more complete models of the vibration system to achieve more pragmatic results, and (3) employ other fuzzy systems (e.g., Takagi-Sugeno and fuzzy type II systems)

to enhance the flexibility of the controller design process.

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