



## Engine Saturation Effect on Consensus of Decentralized Bi-Directional Nonlinear Self-Driving Vehicle Convoys

Hossein Chehardoli \*

Ayatollah Boroujerdi University, Department of Mechanical Engineering, Boroujerd, Iran

### ARTICLE INFO

#### Article history:

Received : 5 Feb 2023

Accepted: 25 Mar 2023

Published: 29 Mar 2023

#### Keywords:

Self-driving vehicle convoy (SDVC)

Engine saturation

Bi-directional

Lyapunov function

Second-order nonlinear SDVs

### ABSTRACT

In this paper, the consensus of second-order nonlinear self-driving vehicle convoys (SDVCs) is studied. We assume that each self-driving vehicle (SDV) communicates only with one front and one rear SDVs. Each SDV's nonlinear dynamics consisting of the rolling resistance and the air drag force is a function of SDV's speed and is investigated in SDVC's modeling and consensus design. Since the speed is bounded, all vehicles' nonlinearities are also bounded. Due to engine saturation of each SDV, the control input is limited. We involve this limitation by introducing the  $\arctan(\cdot)$  function to control protocol. The inter-SDV's distances are assumed to be constant during motion. The distance tracking error associated with each SDV is defined as distance between it and the leading SDV. The error dynamics of the proposed SDVC is derived after applying the consensus law to each SDV. To prove the internal stability, the Lyapunov theorem is employed. We will prove that under this consensus algorithm, the SDVC will be internal stable. To validate the effectiveness of this method, a SDVC comprising a leading and 6 following SDVs will be studied. It will be verified that under the proposed consensus law, all the SDVs reach a unique consensus.

## 1. Introduction

In recent decades, we have perceived significant progress in controlling the motion of self-driving vehicles (SDVs) [1-3]. Self-driving vehicle convoys (SDVCs) have played a very immense role in creating and making intelligent traffic flows [4, 5]. The consensus as an important problem of SDVCs is investigated by many researchers [6-8]. We say that a SDVC achieves consensus if all SDVs reach a same speed and acceleration with a safe constant distance between consecutive SDVs [9].

The distance between SDVs can be fixed or variable. If constant, the length of the SDVC always remains constant, but if this distance is a function of the convoy speed, the length increases during acceleration and shortens during braking. If the interval between SDVs is always constant, the traffic capacity will be higher than when it is time-varying, although its practical implementation is more difficult [10, 11].

If the consensus has a unique solution, the SDVC is called internally stable. Moreover, if the distance error range does not increase among the

\*Corresponding Author

[hchehardoli@gmail.com](mailto:hchehardoli@gmail.com), [h.chehardoli@abru.ac.ir](mailto:h.chehardoli@abru.ac.ir)

<http://doi.org/10.22068/ase.2023.627>

## Engine Saturation Effect on Consensus of Decentralized Bi-Directional Nonlinear Self-Driving Vehicle Convoys

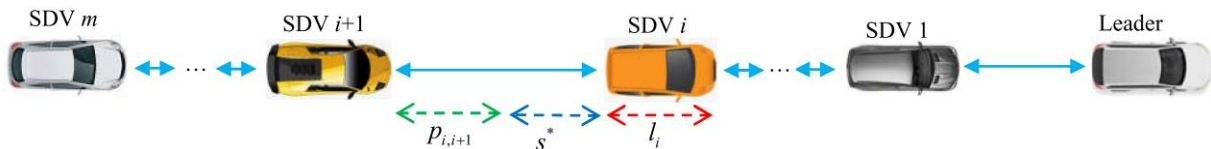
SDVs, the SDVC is called string stable [12]. According to how information is exchanged between SDVs, there are several communication structures in SDVCs. Directed structures such as centralized and decentralized predecessor following [13, 14], bi-directional [15] and multi-SDVs following [16].

Nonlinear dynamics and engine saturation have significant effects on stability of SDVCs. Due to several effects such as rolling resistance between tires and the road, power transmission structure and air resistant force the nonlinearities appear frequently in upper level dynamics of each SDV [17]. Nonlinearities may cause internal instability of SDVCs. Due to the limited speed of each SDV, all these nonlinear terms will be bounded. On the other hand, due to structural limitations of engine, the controller will be bounded. Few works have been carried out on internal stability of SDVCs with dynamical nonlinearities. A nonlinear consensus based on parameter identification is proposed in [18] for different topologies. A robust backstepping method to compensate the nonlinear dynamics is presented in [17]. Optimal consensus design in the presence of actuator delay and nonlinearities is performed in [19]. A nonlinear hierarchical model predictive approach to achieve the consensus and collision prevention of SDVCs is introduced in [20]. Effect of actuator fault on the stability of SDVCs is studied in [9] and [21]. The internal stability under input saturation, parameter uncertainty and time-varying distances is investigated in [22]. Adaptive robust finite-time consensus with unknown saturation and bound disturbance is presented in [23]. Neural network-based estimation design of uncertain SDVCs under

input saturation and nonlinear uncertainties is studied in [24]. The comfort and safety problems in the presence of input saturation, time delay and time-varying distances are proposed in [10]. In [25], the effects of actuator fault and saturation on convoy motion on uneven surfaces are investigated.

In the previous works, the consensus problem of second-order bi-directional (BD) decentralized SDVCs in the presence of nonlinearity and engine saturation has not been investigated. Therefore, we will solve the consensus problem of BD decentralized nonlinear second-order SDVCs in the presence of engine saturation. The motion of the leading SDV is known and all following SDVs' motion is described by second-order nonlinear differential models. The nonlinearities are caused by the rolling resistance and air forces. Therefore, all these terms are bounded by an  $\arctan(\cdot)$  function. Due to engine saturation, the control input of each SDV is limited. This limitation is modeled by the  $\arctan(\cdot)$  in the consensus law. All distances between SDVs are designed to be constant. The distance error of each following SDV with respect to leading SDV is defined and the dynamics of the closed-loop of the SDVC is obtained according to error dynamics. To obtain the consensus of the whole system, a Lyapunov function is defined. It will be proved that under the proposed consensus law, the BD decentralized SDVC with engine saturation will be internal stable. To verify the effectiveness of this method, numerical simulations are provided.

We arrange the remain of this article as below. In part 2, the problem is introduced and useful mathematical tools are presented. In part 3, the



**Figure 1.** A SDVC with bi-directional topology

consensus design procedure is presented. Part 4 provides the numerical studies and part 5 concludes the results and offers future guidelines.

## 1. System definition

Consider a SDVC of  $m$  SDVs and a leader with bi-directional topology as shown in figure 1. Leader motion is formulated by:

$$\ddot{p}_0(t) = \phi_0(t) \Rightarrow \begin{cases} \dot{p}_0(t) = q_0(t) \\ \dot{q}_0(t) = f_0(t) \end{cases} \quad (1)$$

where  $p_0(t)$ ,  $q_0(t)$  and  $f_0(t)$  are position, speed and the known function associated with the leader. Each of the SDV is formulated according to a second-order nonlinear equation as follows:

$$\begin{aligned} \ddot{p}_i(t) &= -(\alpha_{1,i} + \alpha_{2,i}q_i) / m_i - \alpha_{3,i}q_i^2 / m_i + u_i^* / m_i \\ \Rightarrow \begin{cases} \dot{p}_i(t) = q_i(t) \\ \dot{q}_i(t) = -(\alpha_{1,i} + \alpha_{2,i}q_i) / m_i - \alpha_{3,i}q_i^2 / m_i + u_i^* / m_i \end{cases} \end{aligned} \quad (2)$$

where  $\alpha_{1,i}$ ,  $\alpha_{2,i}$  are rolling resistance coefficients,  $m_i$  is the mass,  $u_i^*$  is consensus protocol and  $\alpha_{3,i}$  is a constant containing air drag coefficient and the geometry of the  $i$ -th SDV. By defining that  $f_i(q_i, t) = -(\alpha_{1,i} + \alpha_{2,i}q_i + \alpha_{3,i}q_i^2)$  and  $u_i = u_i^* / m_i$ , (2) will be rewritten as

$$\begin{cases} \dot{p}_i(t) = q_i(t) \\ \dot{q}_i(t) = f_i(t, q_i) + u_i \end{cases} \quad (3)$$

The consensus of the SDVC (1) and (3) is achieved if we have:

$$\begin{cases} \lim_{t \rightarrow \infty} |p_{i-1}(t) - p_i(t) - s^* - l_{i-1}| = 0 \\ \lim_{t \rightarrow \infty} |q_{i-1}(t) - q_i(t)| = 0 \end{cases}, \quad i = 1, 2, \dots, m \quad (4)$$

where  $s^*$  is the safe distance and  $l_k$  is the  $k$ -th SDV length, respectively.

**Lemma 1:** mean value theorem [26]. Suppose that  $h: [t_1, t_2] \rightarrow \mathbb{R}$  is a continuous function. There exists a constant  $t_1 < t^* < t_2$  such as:

$$\int_{t_1}^{t_2} h(\tau) d\tau = h(t^*)(t_2 - t_1) \quad (5)$$

**Assumption 1.** In practical implementations, the velocity of each SDV is bounded. Therefore, we can assume that

$$|f_i(t, q_i)| \leq c_i |\arctan(q_i)| \quad (6)$$

where  $c_i$  is a positive constant.

## 2. Consensus protocol design

To achieve the consensus in the presence of engine's saturation, the below consensus law is designed for each SDV.

$$u_i = \arctan(p_{i-1,i}) + \arctan(p_{i,i+1}) - \alpha_i \arctan(q_i) \quad (7)$$

where  $\alpha_i$  is a positive gain and:

$$\begin{aligned} p_{i-1,i} &= p_{i-1} - p_i - s^* - l_{i-1} \\ p_{i,i+1} &= p_{i+1} - p_i + s^* + l_i \end{aligned} \quad (8)$$

From (7), we can infer that:

$$|u_i(t)| \leq \pi \left( 1 + \frac{1}{2} \alpha_i \right) \quad (9)$$

By applying (7) to (3), the  $i$ -th SDV's closed-loop dynamics is obtained as below:

$$\begin{cases} \dot{p}_i(t) = q_i(t) \\ \dot{q}_i(t) = f_i(t, q_i) + \arctan(p_{i-1,i}) + \arctan(p_{i,i+1}) - \alpha_i \arctan(q_i) \end{cases} \quad (10)$$

We define that:

$$\begin{aligned} \xi_i &= \arctan(p_{i-1,i}) + \arctan(p_{i,i+1}), \xi = [\xi_1, \xi_2, \dots, \xi_m]^T \\ \alpha &= [\alpha_1, \alpha_2, \dots, \alpha_m]^T, \mathbf{p} = [p_1, p_2, \dots, p_m]^T, \mathbf{q} = [q_1, q_2, \dots, q_m]^T \\ \bar{q}_i &= \arctan(q_i), \bar{\mathbf{q}} = [\bar{q}_1, \bar{q}_2, \dots, \bar{q}_m]^T, \mathbf{f} = [f_1, f_2, \dots, f_m]^T \end{aligned}$$

Therefore, (10) is rewritten as:

$$\begin{cases} \dot{\mathbf{p}}(t) = \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) = \xi(t) - \alpha \bar{\mathbf{q}}(t) + \mathbf{f}(t) \end{cases} \quad (11)$$

Theorem 1 solves the consensus problem.

**Theorem 1.** If the parameter  $\alpha_i$  satisfies the following constraint, the SDVC described through (1) and (3) under the consensus law (7) and assumption 1 will obtain the consensus.

$$\alpha_i > c_i \quad (12)$$

## Engine Saturation Effect on Consensus of Decentralized Bi-Directional Nonlinear Self-Driving Vehicle Convoys

**Proof.** To prove, the below Lyapunov function is defined.

$$V = \sum_{i=1}^m \left( \int_0^{p_{i-1,i}} \arctan(\tau) d\tau + \int_0^{p_{i+1,i}} \arctan(\tau) d\tau \right) + \sum_{i=1}^m q_i^2 \quad (13)$$

According to lemma 1, there exists a positive value  $0 < \mu_i < p_{k,i}$  satisfying that  $\int_0^{p_{k,i}} \arctan(\tau) d\tau = \arctan(\mu_i) p_{k,i} > 0$  where  $k$  can be  $i-1$  or  $i+1$ .

Time differentiating of (13) along (11) yields

$$\begin{aligned} \dot{V} &= -2 \sum_{i=1}^m q_i (\arctan p_{i-1,i} + \arctan p_{i+1,i}) + 2 \sum_{i=1}^m q_i \dot{q}_i \\ &= -2 \mathbf{q}^T \dot{\xi} + 2 \mathbf{q}^T \dot{\mathbf{q}} \\ &= -2 \mathbf{q}^T (\dot{\mathbf{q}} + \mathbf{a}\bar{\mathbf{q}} - \mathbf{f}) + 2 \mathbf{q}^T \dot{\mathbf{q}} \\ &= -2 \mathbf{q}^T \dot{\mathbf{q}} - 2 \mathbf{q}^T \mathbf{a}\bar{\mathbf{q}} + 2 \mathbf{q}^T \mathbf{f} + 2 \mathbf{q}^T \dot{\mathbf{q}} \\ &= -2 \mathbf{q}^T \mathbf{a}\bar{\mathbf{q}} + 2 \mathbf{q}^T \mathbf{f} \\ &= -2 \sum_{i=1}^m \alpha_i q_i \arctan q_i + 2 \sum_{i=1}^m q_i f_i(t, q_i) \end{aligned} \quad (14)$$

According to assumption 1, one can write

$$\begin{aligned} \sum_{i=1}^m q_i f_i(t, q_i) &\leq \sum_{i=1}^m |q_i| \cdot |f_i(t, q_i)| = \\ &= \sum_{i=1}^m c_i q_i \arctan q_i \end{aligned} \quad (15)$$

Therefore,

$$\begin{aligned} \dot{V} &\leq -2 \sum_{i=1}^m \alpha_i q_i \arctan q_i + 2 \sum_{i=1}^m c_i q_i \arctan q_i \\ &= -2 \sum_{i=1}^m (\alpha_i - c_i) q_i \arctan q_i \end{aligned} \quad (16)$$

and under (12), we have  $\dot{V} \leq 0$ .

According to (16), it is inferred that when  $\dot{V} = 0$  we have  $q_i = 0$ . From (10), we have

$$f_i(t, q_i) + \arctan(p_{i-1,i}) + \arctan(p_{i+1,i}) = 0 \quad (17)$$

From assumption 1, we have  $|f_i(t, q_i)| \leq c_i |\arctan(q_i)| = 0$  and

$$\begin{aligned} f_i(t, q_i) = 0 \Rightarrow \\ \arctan(p_{i-1,i}) + \arctan(p_{i+1,i}) = 0 \end{aligned} \quad (18)$$

So that,

$$\begin{aligned} \sum_{i=1}^m (p_i - s^* - l_{i-1}) \arctan(p_{i-1,i}) &= 0, \\ \sum_{i=1}^m p_{i-1,i} \arctan(p_{i-1,i}) &= 0 \end{aligned} \quad (19)$$

By subtracting the above equations and knowing that, the network topology is bi-directional and therefore, symmetric, we will have:

$$\sum_{i=1}^m p_{i-1,i} \arctan(p_{i-1,i}) = 0 \quad (20)$$

We know that for  $\mathcal{G} \neq 0$ :  $\mathcal{G} \arctan(\mathcal{G}) > 0$ . Therefore, from (20) we infer that  $p_{i-1,i} = 0$  and according to (4) the consensus is achieved and the proof is complete.

**Remark 1.** Theorem 1 is presented for bi-directional network topology. This approach can be applied to all symmetric networks.

In the following, we present a comparison for a case where engine saturation is not considered. For this case, the consensus protocol (7) is redesigned as follows:

$$u_i = p_{i-1,i} + p_{i+1,i} - \bar{c}_i q_i \quad (21)$$

where  $\bar{c}_i$  is a positive gain. In the case that the input has no saturation, assumption 1 is modified as follows.

**Assumption 2.** Without considering saturation, the nonlinear function  $f_i(t, q_i)$  is bounded by

$$|f_i(t, q_i)| \leq c_i^* |q_i| \quad (22)$$

where  $c_i^*$  is a positive constant. Under (21), (10) will be as below:

$$\begin{cases} \dot{p}_i(t) = q_i(t) \\ \dot{q}_i(t) = f_i(t, q_i) + p_{i-1,i} + p_{i+1,i} - \bar{c}_i q_i \end{cases} \quad (23)$$

By defining  $\bar{\xi}_i = p_{i-1,i} + p_{i+1,i}$ ,  $\bar{\xi} = [\bar{\xi}_1, \bar{\xi}_2, \dots, \bar{\xi}_m]^T$  and  $\bar{\mathbf{C}} = \text{diag}(\bar{c}_1, \bar{c}_2, \dots, \bar{c}_m)$ , (11) will be as follows

$$\begin{cases} \dot{\mathbf{p}}(t) = \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) = \bar{\xi}(t) - \bar{\mathbf{C}}\mathbf{q}(t) + \mathbf{f}(t) \end{cases} \quad (24)$$

Now, we modify theorem 1 as follows.

**Theorem 2.** The SDVC described through (1) and (3) with unsaturated engine's input, under the consensus law (21) and assumption 2 will reach to consensus if we have:

$$\bar{c}_i > c_i^* \quad (25)$$

**Proof.** To prove the consensus problem, the following Lyapunov function is defined.

$$V = \frac{1}{2} (p_{i-1,i}^2 + p_{i+1,i}^2) + \sum_{i=1}^m q_i^2 \quad (26)$$

Differentiating (26) yields

$$\begin{aligned} \dot{V} = & \sum_{i=1}^m [(q_{i-1} - q_i) p_{i-1,i} + (q_i - q_{i+1}) p_{i+1,i}] + \\ & + 2 \sum_{i=1}^m \dot{q}_i q_i \end{aligned} \quad (27)$$

By simplifying, (27) can be written in the following compact form

$$\begin{aligned} \dot{V} = & -2\mathbf{q}^T \bar{\xi} + 2\mathbf{q}^T \dot{\mathbf{q}} \\ = & -2\mathbf{q}^T (\dot{\mathbf{q}} + \bar{\mathbf{C}}\mathbf{q} - \mathbf{f}) + 2\mathbf{q}^T \dot{\mathbf{q}} \\ = & -2\mathbf{q}^T \bar{\mathbf{C}}\mathbf{q} + 2\mathbf{q}^T \mathbf{f} = -2 \sum_{i=1}^m \bar{c}_i q_i^2 + 2 \sum_{i=1}^m q_i f_i(t, q_i) \end{aligned} \quad (28)$$

According to assumption 2, one can write

$$\sum_{i=1}^m q_i f_i(t, q_i) \leq \sum_{i=1}^m |q_i| \cdot |f_i(t, q_i)| \leq \sum_{i=1}^m c_i^* q_i^2 \quad (29)$$

Consequently,

$$\dot{V} \leq -2 \sum_{i=1}^m \bar{c}_i q_i^2 + 2 \sum_{i=1}^m c_i^* q_i^2 \leq -2 \sum_{i=1}^m (\bar{c}_i - c_i^*) q_i^2 \leq 0 \quad (30)$$

According to (30), it is inferred that when  $\dot{V} = 0$  we have  $q_i = 0$ . From (23), we have

$$f_i(t, q_i) + p_{i-1,i} + p_{i+1,i} = 0 \quad (31)$$

From assumption 2, we have  $|f_i(t, q_i)| \leq c_i^* |q_i| = 0$ . So that

$$f_i(t, q_i) = 0 \Rightarrow p_{i-1,i} + p_{i+1,i} = 0 \quad (32)$$

Accordingly,

$$\sum_{i=1}^m p_{i-1,i} p_{i+1,i} = 0 \quad (33)$$

The network topology is symmetric. Therefore,

$$\begin{aligned} \sum_{i=1}^m p_{i-1,i}^2 &= \sum_{i=1}^m p_{i-1,i} p_{i-1} - \sum_{i=1}^m p_{i-1,i} p_i = \\ &= 2 \sum_{i=1}^m p_{i-1,i} p_{i-1} = 0 \end{aligned} \quad (34)$$

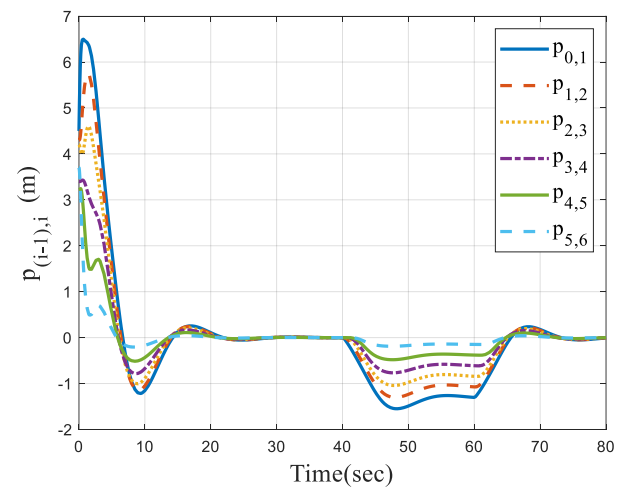
which indicates that  $p_{i-1,i}$ ,  $i = 1, 2, \dots, m$  and the consensus has a unique solution.

**Remark 2.** It should be noted that the coefficients  $\alpha_{1,i}$ ,  $\alpha_{2,i}$  and  $\alpha_{3,i}$  are small. When they are divided by  $m_i$ , since the velocity is bounded, the resultant nonlinear term  $f_i(q_i, t)$  will be smaller and therefore, the assumptions (6) and (22) are reasonable.

### 3. Verification study

To verify the performance of the proposed consensus algorithm in theorem 1, a convoy comprising 7 SDVs is considered. We perform the simulation for two braking and accelerating maneuvers. The constants are supposed as:

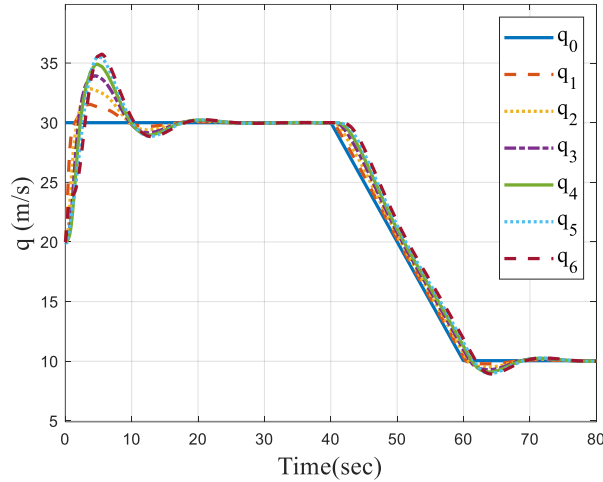
$$\begin{aligned} s^* &= 5m, \alpha_i = 4.6, \bar{c}_i = 4.1, m_1 = 1400kg, m_2 = 1500kg, \\ m_3 &= 1350kg, m_4 = 1450kg, m_5 = 1410kg, m_6 = 1440kg, \\ l_1 &= 3.5m, l_2 = 3.8m, l_3 = 4.2m, l_4 = 4.4m, l_5 = 4.3m, \\ l_6 &= 3.8m. \end{aligned}$$



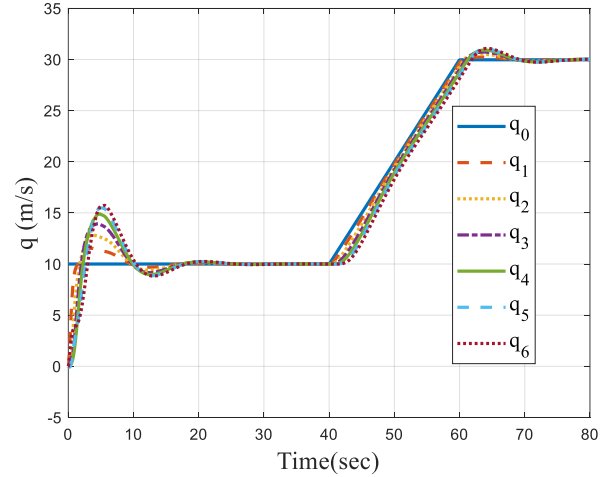
**Figure 2:**  $p_{i-1,i}$  for the convoy: braking maneuver

Figures 2 and 5 illustrate the distance error of the SDVC in braking and accelerating maneuvers, respectively. As these figures show, all distance tracking errors tend to zero specifying internal stability in both acceleration and braking maneuvers. Figures 3 and 6 depict the velocity and figures 4 and 7 show the acceleration of SDVs in braking and accelerating maneuvers, respectively. Since the SDVC is internal stable, the following SDVs track the speed and acceleration of leading SDV.

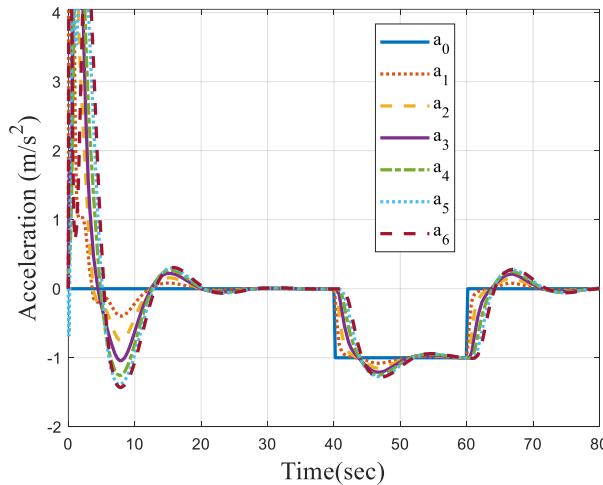
# Engine Saturation Effect on Consensus of Decentralized Bi-Directional Nonlinear Self-Driving Vehicle Convoys



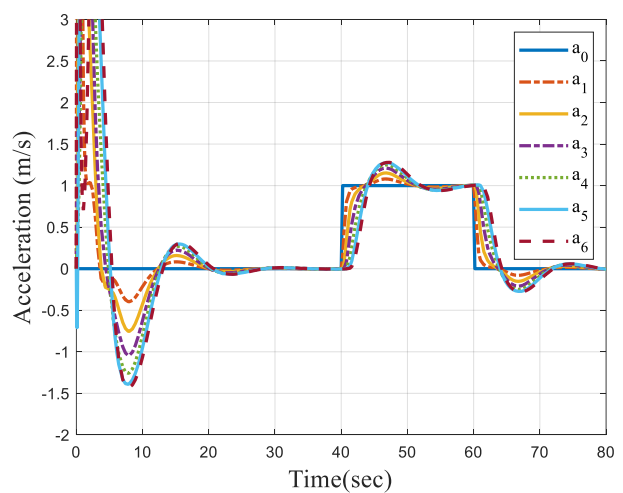
**Figure 3:** Speed of the convoy: braking maneuver



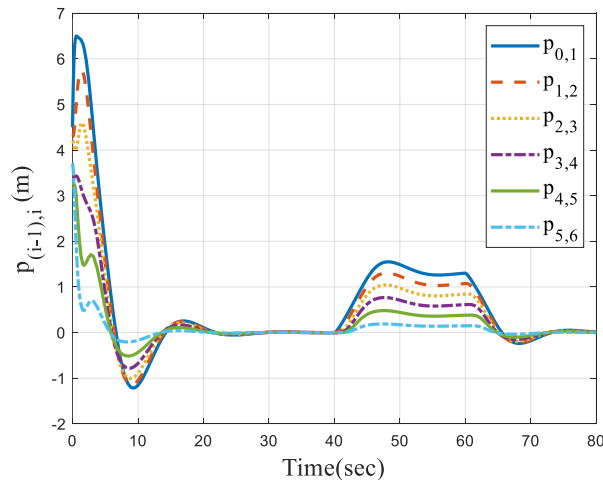
**Figure 6:** Speed of the convoy: accelerating maneuver



**Figure 4:** Acceleration of the convoy: braking maneuver



**Figure 7:** Acceleration of the convoy: accelerating maneuver



**Figure 5:**  $p_{i-1,i}$  for the convoy: braking maneuver

## 4. Conclusions

The consensus of second-order nonlinear SDVCs with bi-directional topology in the presence of engine saturation was studied. Each SDV's nonlinear dynamics consisting of the rolling resistance and the air drag force is a function of SDV's speed. Due to engine saturation, the control input is limited. We involved this limitation by introducing the  $\arctan(\cdot)$  function to control protocol. The error dynamics of the proposed SDVC was derived after applying the consensus law to each SDV. To prove the internal stability, the second Lyapunov theorem was employed. It was shown that under this consensus algorithm, the SDVC is internal stable. To verify the effectiveness of this method,

a SDVC comprising a leading and 6 following SDVs was studied. The obtained results showed the merits of the proposed method. For future works, the engine time constant can be added to the presented approach of this paper.

## References

- [1] C. Bergenheim, S. Shladover, E. Coelingh, C. Englund, and S. Tsugawa, "Overview of platooning systems," in *Proceedings of the 19th ITS World Congress, Oct 22-26, Vienna, Austria (2012)*, 2012.
- [2] H. Ge, R. Cheng, and L. Lei, "The theoretical analysis of the lattice hydrodynamic models for traffic flow theory," *Physica A: Statistical Mechanics and its Applications*, vol. 389, no. 14, pp. 2825-2834, 2010.
- [3] S. Santini, A. Salvi, A. S. Valente, A. Pescapé, M. Segata, and R. L. Cigno, "A consensus-based approach for platooning with intervehicular communications and its validation in realistic scenarios," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 3, pp. 1985-1999, 2016.
- [4] L. D. Baskar, B. De Schutter, and H. Hellendoorn, "Optimal routing for automated highway systems," *Transportation Research Part C: Emerging Technologies*, vol. 30, pp. 1-22, 2013.
- [5] W. Zhang, E. Jenelius, and H. Badia, "Efficiency of semi-autonomous and fully autonomous bus services in trunk-and-branches networks," *Journal of Advanced Transportation*, vol. 2019, 2019.
- [6] C. Deng and G.-H. Yang, "Leaderless and leader-following consensus of linear multi-agent systems with distributed event-triggered estimators," *Journal of the Franklin Institute*, vol. 356, no. 1, pp. 309-333, 2019.
- [7] Y. Zheng, Q. Zhao, J. Ma, and L. Wang, "Second-order consensus of hybrid multi-agent systems," *Systems & Control Letters*, vol. 125, pp. 51-58, 2019.
- [8] P. Yang, Y. Ding, X. Hu, Z. Zhang, and Z. Wang, "Sliding mode fault-tolerant consensus control for heterogeneous multi-agent systems based on finite-time observer and controller," *Transactions of the Institute of Measurement and Control*, p. 01423312221150292, 2023.
- [9] G. Guo, P. Li, and L.-Y. Hao, "Adaptive fault-tolerant control of platoons with guaranteed traffic flow stability," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 7, pp. 6916-6927, 2020.
- [10] J. Chen, H. Liang, J. Li, and Z. Lv, "Connected automated vehicle platoon control with input saturation and variable time headway strategy," *IEEE Transactions on Intelligent Transportation Systems*, vol. 22, no. 8, pp. 4929-4940, 2020.
- [11] Y. Bian, Y. Zheng, W. Ren, S. E. Li, J. Wang, and K. Li, "Reducing time headway for platooning of connected vehicles via V2V communication," *Transportation Research Part C: Emerging Technologies*, vol. 102, pp. 87-105, 2019.
- [12] X. Guo, J. Wang, F. Liao, and R. S. H. Teo, "Distributed adaptive integrated-sliding-mode controller synthesis for string stability of vehicle platoons," *IEEE Transactions on Intelligent Transportation Systems*, vol. 17, no. 9, pp. 2419-2429, 2016.
- [13] P. Wijnbergen and B. Besselink, "Existence of decentralized controllers for vehicle platoons: On the role of spacing policies and available measurements," *Systems & Control Letters*, vol. 145, pp. 1-9, 2020.
- [14] J. Wang, X. Luo, W. Wong, and X. Guan, "Specified-Time Vehicular Platoon Control With Flexible Safe Distance Constraint," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 11, pp. 10489-10503, 2019, doi: 10.1109/TVT.2019.2939558.
- [15] I. Herman, S. Knorn, and A. Ahlén, "Disturbance scaling in bidirectional vehicle platoons with different asymmetry in position and velocity coupling," *Automatica*, vol. 82, pp. 13-20, 2017.
- [16] M. Di Bernardo, A. Salvi, and S. Santini, "Distributed consensus strategy for platooning of vehicles in the presence of time-varying heterogeneous communication delays," *IEEE Transactions on Intelligent Transportation Systems*, vol. 16, no. 1, pp. 102-112, 2014.
- [17] J. Huang, Q. Huang, Y. Deng, and Y.-H. Chen, "Toward robust vehicle platooning with bounded spacing error," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 36, no. 4, pp. 562-572, 2016.
- [18] H. Chehardoli and A. Ghasemi, "Adaptive centralized/decentralized control and identification of 1-D heterogeneous vehicular platoons based on constant time headway policy," *IEEE Transactions on Intelligent Transportation Systems*, vol. 19, no. 10, pp. 3376-3386, 2018.

- [19] W. Yue and G. Guo, "Guaranteed cost adaptive control of nonlinear platoons with actuator delay," *Journal of dynamic systems, measurement, and control*, vol. 134, no. 5, pp. 136-144, 2012.
- [20] W.-J. Liu, H.-F. Ding, M.-F. Ge, and X.-Y. Yao, "Cooperative control for platoon generation of vehicle-to-vehicle networks: a hierarchical nonlinear MPC algorithm," *Nonlinear Dynamics*, vol. 108, no. 4, pp. 3561-3578, 2022.
- [21] L.-Y. Hao, P. Li, and G. Guo, "String stability and flow stability for nonlinear vehicular platoons with actuator faults based on an improved quadratic spacing policy," *Nonlinear Dynamics*, vol. 102, pp. 2725-2738, 2020.
- [22] Y. He, X. Tian, J. Shen, C. Yuan, and Y. Du, "Robust stabilization of longitudinal tracking for cooperative adaptive cruise control considering input saturation," *Modern Physics Letters B*, vol. 34, no. 35, p. 2050409, 2020.
- [23] Z. Gao, Y. Zhang, and Q. Liu, "Adaptive finite-time cooperative platoon control of connected vehicles under actuator saturation," *Asian Journal of Control*, vol. 24, no. 6, pp. 3552-3565, 2022.
- [24] X.-G. Guo, J.-L. Wang, F. Liao, and R. S. H. Teo, "CNN-based distributed adaptive control for vehicle-following platoon with input saturation," *IEEE Transactions on Intelligent Transportation Systems*, vol. 19, no. 10, pp. 3121-3132, 2017.
- [25] C. Pan, Y. Chen, Y. Liu, and I. Ali, "Adaptive resilient control for interconnected vehicular platoon with fault and saturation," *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 8, pp. 10210-10222, 2021.
- [26] G. B. Thomas, M. D. Weir, and J. Hass, "Thomas' Calculus: Multivariable," 2010.