Nonlinear Predictive Planning and Control of the Turn-around Task in Autonomous Vehicles

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	The turn-around task is one of the challenging maneuvers in automated driving which requires intricate decision making, planning and control, concomitantly. During automatic turn-around maneuver, the path curvature is too large which makes the constraints of the system severely restrain the path tracking performance. This paper highlights the path planning and control design for single and multi-point turn of autonomous vehicles. The preliminaries of the turn-around task including environment, vehicle modeling, and equipment are described. Then, a predictive approach is proposed for planning and control of the vehicle. In this approach, by taking the observation of the road and vehicle conditions into account and considering the actuator constraints in cost function, a decision is made regarding the minimum number of steering to execute turn-around. The constraints are imposed on the speed, steering angle, and their rates. Moreover, the collision avoidance with road boundaries is developed based on the GJK algorithm. According to the simulation results, the proposed system adopts the minimum number of appropriate steering commands while incorporating the constraints of the actuators and avoiding collisions. The findings demonstrate the good performance of the proposed approach in both path design and tracking for single- and multi-point turns.

1. Introduction

During the past decade, the automotive industry has made significant progress in producing safe, comfortable, and cost-effective vehicles. Studies suggest that the areas of environment, safety, and passenger comfort are critical problems for automotive companies [1]. In the fields of planning, control, and decision-making, turning mission is recognized as one of the sophisticated scenarios. [2]. A special case of this mission is the turn-around task. In this operation on two-way roads, the car tries to place in the opposite lane with a complete change of the vehicle heading angle,

i.e., 180º. This mission is vital in the circumstances involving emergencies, unanticipated events, or passenger demands. Accomplishing this task on narrow and wide roads have different issues and bring difficulties, especially for non skil drivers, such that improper execution of multi-point turn maneuver can damage vehicle body. Applying the automatic turn-around system, makes the mission safer, faster, and easier for all road users.

In recent years, few attempts in the academic and industrial literature have been made to investigate the turning mission and its various aspects including decision making, planning, and control

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[3-5]. There are ongoing projects in this field as a subset of autonomous driving, especially in major automotive technology developers such as Google Car. As a pioneer, Google refers to this mission as one of the most complex driving maneuvers, even for experienced drivers [2].

Model predictive control (MPC) is one of the most popular techniques in the field of autonomous vehicle (AV). Thanks to the extensive improvement of the hardware technologies in computer sciences and also the extensive research done in the theory of optimal control, MPC is able to overcome challenges arising in computational burden, optimality, and applying more physical constraints of the real-world problems [6-10]. In the literature of AVs, the predictive strategy is used for decision making, motion control, planning and also the combination of the mentioned tasks.

The purpose of motion control is to make the AV follow a reference trajectory while considering input constraints during the motion. In [11], an adaptive Multi-MPC is used to the task of path tracking, which benefits from weight adaptation to be utilized in different driving scenarios. In [12], a MPC is hired to fulfil the task of path following and yaw stability using torque vectoring, in high speed lane-changes. In [13, 14], the problem of motion control for the task of cut-in of autonomous vehicles is investigated by the MPC. The prediction task is done using long short term memory networks and lateral velocity control is done by the MPC [13]. Also, the task of cut-in in different scenarios is investigated using MPC, along with the prediction of cut-in using a rule-based method [14]. Besides, in [15] a nonlinear predictive control (NMPC) is investigated that includes the trajectory tracking of a path with waypoints in the presence of modelling uncertainties, guarantying the robustness of the NMPC controller. In addition, MPC strategy is hired for many other challenging cases of motion control, including the high accuracy prediction of changing velocity effect [16], learning-based MPC for race car control [17], and low computational burden Takagi-Sugeno MPC [18].

The aim of motion planning is to design a safe and feasible trajectory to be tracked by the controller, and also the task of decision-making is considered. In [19], a decision-making framework based on MPC is used to not only human-like decision process, but also for state prediction and collision avoidance path planning. In [20], Social behavior of the traffic participants and smooth decision-making of autonomous vehicles in the

task of lane-change planning is considered using MPC. The design of a safe and maneuverable path in the head-on collision scenario is carried out with the help of MPC method in [21], which includes the uncertainties in the motion of the vehicle that has deviated from the opposite lane. Moreover, there are research works in the literature that use MPC to overcome other challenges like moving object constraints [22], nonlinearity of kinematic model [23], and possible future occupied spaces of the vehicles using reachability robustness analysis [24]. To design an efficient trajectory planning solution capable of spatial-temporal joint optimization, [25] utilized the differential flatness property of car-like robots to real-time trajectory optimization that can generate feasible trajectory under arbitrary constraints.

There are many research in the literature that incorporate both planning and tracking in MPC for an autonomous vehicle. Using non-linear models in the path planning and control system design increases the computational load and leads to unwanted complexity. In [26], By examining the vehicle characteristics, the MPC is designed with less complexity than the non-linear models, which results in less computational load due to its independence from friction. In [27], an MPC control strategy is hired to design the vehicle's lateral motion in curved paths, by considering the dynamic constraints of the vehicle. Furthermore, the designed path is smooth and benefits from continuity, and also the passenger safety challenges have also been taken into account in path following task.

According to the best knowledge of the authors, the nonlinear model predictive control of turn-around task has not been comprehensively studied so far. The contributions of this research are listed as below.

- Considering the constraints include the limit of the steering wheel angle, speed and rate of them make better path tracking performance which are done using model predictive control.
- However, for turn-around task, the accuracy of the linearized prediction model is insufficient. To address this issue, a path tracking controller based on nonlinear model predictive control was proposed in this paper.
- The advantage of the proposed method is that all tasks of decision making, planning and controlling the motion of the autonomous vehicle in the turn-around

maneuver are performed simultaneously, in an integrated form.

- The control-oriented model used in simulations, imposes a low computational load to the processing system, and on the other hand, the validation of the model is carried out with the dynamic behavior of the vehicle.
- The proposed system is investigated in roads with different widths, and is able to decide the required number of steering commands (single or multi-point steering commands) was used to turn-around, planning, and control of the vehicle.

According to the contents, the turn-around task will be described in details, in the next section. Then, the proposed approach, including collision avoidance constraints and nonlinear model predictive control, is presented. In the following, the simulation setup and results are prepared. First, the control-oriented model is validated, then, the simulations are performed for the turn-around task in various scenarios and discussed. Finally, the conclusion section summarizes the whole paper.

2. Description of the turn-around task

2.1. Environment

Figure 1 shows a schematic of single and multi-point turn maneuver on narrow and wide roads. The point P is introduced as the midpoint of the car rear axle. The lateral distance from the midpoint to the road curb in the opposite lane equals a, and $W_a = W_r - W_v$ represents the whole permissible width of the road that restricts the vehicle operational area.

The single-point turn in Figure 1-a, is defined as a turn whose sign of speed does not change during the maneuver. Whereas, the multi-point turns experience at least two change of speed sign, as in Figure 1-b. For instance, three and five-point turns have two and four speed sign change, respectively. For single-point turn, the execution zone is supposed to be adequately large to turn-around with a single maneuver. After the vehicle reaches the opposite lane, it is placed at a safe distance from the right side of the road, so as not to violate the smooth flow of the traffic.

Although U-shaped turning is more desirable than other turning maneuvers, it may not be possible in specific driving scenarios. This occurs when the geometry of the road and its allowable
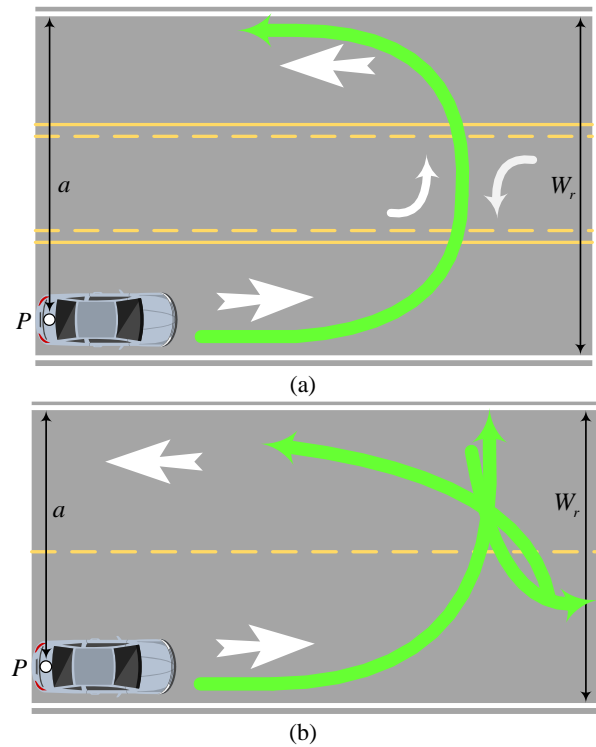

(a)

(b)

Figure 1: Schematic of the turn-around task, (a) Single-point turn (U-turn) on the wide road, (b) Three-point turn on the narrow road.

width for reversing the vehicle heading are less than the minimum width of the single-point turn, W_{1min}. However, if the road still has the capacity that the vehicle reverse its heading, the task of turning around can be performed by several maneuvers, W_{Nmin}. The best decision should be made to choose the least number of commands, depending on the situation. A standard recommendation to start this task is to first the vehicle moves to a legal position parallel to the road boundary with a lane change-stop maneuver [28]. Then, the car must make a chain of back-and-forth movements until it accomplish the mission.

2.2. Vehicle model

In turn-around task, the vehicle travels at low speeds. Therefore, the Ackermann steering is considered rolling around the instantaneous center of rotation without slipping. In this study, the vehicle has two axles with its bounding rectangle geometry in order to consideration of collision avoidance with road curbs. A schematic of the dual-track vehicle model is shown in Figure 2 and related parameter is described in Table 1.
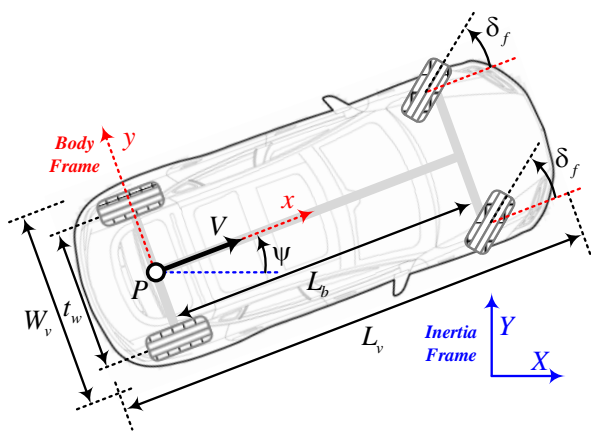
Figure 2: Layout of the four-wheel vehicle model, the inertia and body frames.

Table 1: Description of vehicle model parameters.

Description	Parameter
Global coordinates	X, Y
Heading angle	ψ
Front wheel steering angle	δ_f
Vehicle speed	V
Track width	t_w
Wheelbase	L_b
Vehicle width	W_v
Vehicle length	L_v

Both full steering angles to left and right are assumed to be equal and related to the smallest radius R_{lock}.

This study utilize a kinematic bicycle model with front steering angle. Therefore, the motion of the vehicle is described using the nonlinear model in Equation (1), [29].

$$\dot{X} = V \cos \psi$$
$$\dot{Y} = V \sin \psi \qquad (1)$$
$$\dot{\psi} = (V/L_b) \tan \delta_f$$

Here, (X, Y) stand for the position of the vehicle and ψ refers to its yaw angle. (X, Y, ψ) are the state variables for the vehicle state functions. The speed V and steering angle δ_f are the control inputs for the vehicle state functions.

2.3. Equipment

The perception module determines the current pose of the car, the dimension of the road, the turning location status, and possible obstacles. This module also prepares the required information about the environment for the planning and control process. To this end, the internal measuring devices and extrospective or infrastructure sensors can be utilized. In this research, it is assumed that all

environmental information and sensor data are available.

Proposed path planning and control module executes the maneuvers. Today, processors related to throttle, brake, and steering actuators commonly exist in vehicles; although, they may require some additional modifications. In short, acceleration is supplied by the engine control unit (ECU). The braking performance is modified using an electronic stability program (ESP) or anti-lock braking system (ABS). Steering of the vehicle is also executed using the steering column and an electronic steering processor.

3. Proposed approach

The overall structure of the proposed integrated planning and control system to perform the turn-around task is presented in Figure 3. In this framework, all the modules including decision making, planning and control are embedded into the nonlinear predictive strategy.

The planning section decides how and when to steer the vehicle towards a single-point or higher-points turn by taking into account the priority of the least number of commands to the vehicle's steering actuator. Furthermore, a prescribed initial guess for the desired path is required in the MPC, which is selected as a straight line connecting the start point to the final point of the maneuver.

It should be noted that the decision and notification of the complete turn has been issued, and we assume that the required information of the traffic condition and the road geometry are provided by a perception system. Our contribution starts with the decision and calculations related to the type of turning maneuver, by checking the feasibility of the turn-around problem.

The MPC controller is hired to calculate the desired path and design the control inputs to follow the mentioned desired path. The components of the planning and control system including the cost function and the terminal costs, collision avoidance constraints, physical constraints of the velocity and steering angle actuators, are described in the following section.

3.1. Collision avoidance constraints

Dynamic safety is necessary to avoid collision of the vehicle with road boundaries during the turning
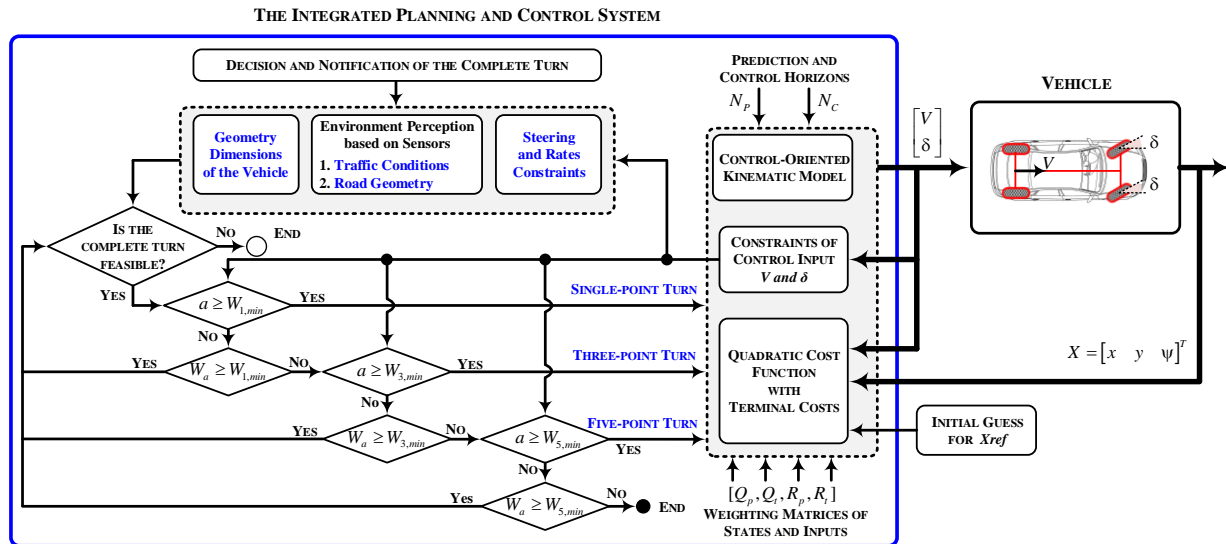
Figure 3: Overall structure of the proposed integrated planning and control system.

around task. The safety conditions are formulated by inequality constraints showing the determined minimum distance between the vehicle corners and road boundaries. To implement these constraints, a signed distance $d_{v,B}$ parameter is defined

$$d_{v,B} = \min_{\rho}\{\|\rho\|_2 : v \oplus \rho \neq \emptyset\} - \min_{\rho}\{\|\rho\|_2 : V \oplus \rho = \emptyset\} \quad (2)$$

Where v and B stand for the vehicle geometry and the boundaries of the road, ρ denotes the translation from v, and $\oplus$ is the sumset. After calculation of $d_{v,B}$ at each moment, the trajectory must be generated in such a way that the distance between the vehicle traveling it and the road boundaries is greater than the safety threshold. The schematic of the collision avoidance constraints are shown in Figure 4.

In order to compute the signed distance, an approximate method is utilized known as a GJK algorithm [25]. This algorithm prepares an efficient computation of signed distances which is suitable for our optimization problem in nonlinear model predictive control method. In this problem, we only need to check each vertex of the vehicle body to get the minimum signed distance. Moreover, the lower bound of a safety distance is predetermined to guarantee collision avoidance.

3.2. Predictive planning & control formulation

Applying the model predictive approach is a good way to handle multiple constraints and large curvature of the reference path. The goal of the vehicle is to turn in narrow and wide roads without

colliding and considering constraint conditions mainly the longitudinal acceleration, speed, steering angle and steering angular velocity of the vehicle.

The core of designing a nonlinear model predictive controller is to establish a predictive model and design an optimization function. In a path-following controller, the function of the predictive model is to predict the possible future position of the vehicle according to the current pose state of the vehicle and possible future control inputs. The output of the vehicle state function is the same as the state of the vehicle, (X, Y, ψ). Therefore, the nonlinear model predictive controller is created with three states, three outputs, and two manipulated inputs.

To establish a prediction model based on the kinematics model of the vehicle, the control-oriented model is written in general form as in equation (3).

$$dx/dt = f(x,u), \quad x = \begin{bmatrix} X \\ Y \\ \psi \end{bmatrix}, \quad u = \begin{bmatrix} V \\ \delta_f \end{bmatrix} \quad (3)$$

In this section, the path planning and tracking controller for automatic turn based on linear time varying model predictive control and the nonlinear one are proposed. It is noticed that the reference point for the vehicle pose is located at the center of rear axle.

The Euler method is utilized for discretizing equation (3), with the current moment t.

$$x(t + 1|t) = x(t|t) + T.f\big(x(t|t), u(t|t)\big) \quad (4)$$

Nonlinear Predictive Planning and Control of the Turn-around Task in Autonomous Vehicles
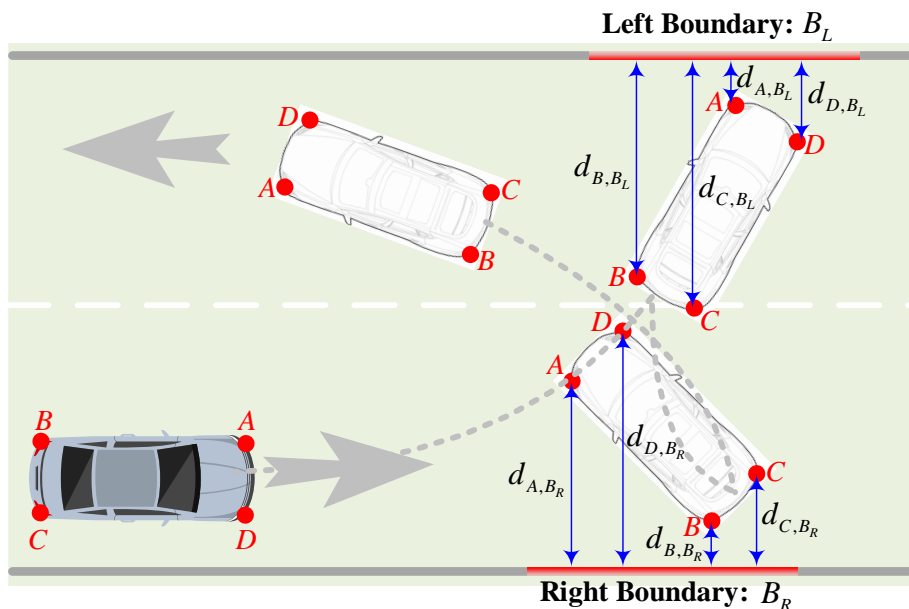

Figure 4: Schematic of the collision avoidance constraints.

Now, defining the prediction and control horizons N_p and N_c, the pose states of the vehicle at each step are calculated as in equation (5).

$$
\begin{aligned}
x(t+1|t) &= x(t|t) + T.f\big(x(t|t), u(t|t)\big) \\
x(t+2|t) &= x(t+1|t) + T.f\big(x(t+1|t), u(t+1|t)\big) \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
x(t+N_c+1|t) &= x(t+N_c|t) + T.f\big(x(t+N_c|t), u(t+N_c|t)\big) \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
x\big(t+N_p|t\big) &= x\big(t+N_p-1|t\big) + T.f\left(x\big(t+N_p-1|t\big), u(t+N_c|t)\right)
\end{aligned}
\tag{5}
$$

The deviation between the vehicle pose and the reference path in the predicted time domain is the error obtained by the prediction.

$$
\begin{aligned}
e(t+i|t) = \\
x(t+i|t) - x_{ref}(t+i|t), \quad i = 1, \dots, N_p
\end{aligned}
\tag{6}
$$

Where x_{ref} denotes the pose state information of the tracking target point on the reference path.

Equations (3) to (6) are the predictive model of the automatic turning path tracking controller. Based on the above model, the nonlinear model predictive controller uses a customized cost function, which is defined in a manner similar to a quadratic tracking cost plus a terminal cost. In the following custom cost function in equation (7),

$x(t)$ denotes the states of the vehicle at time t, the parameter d represents the duration of simulation, and x_{ref} is the target pose of the vehicle.

In equation (7), the first penalty term is an error penalty term, the function of the second penalty term is to make the automatic turning path as smooth as possible, and the rest functions are for the terminal point. In this cost function, $\mathbf{Q}_p$ and $\mathbf{R}_p$ are constant tracking weight matrices, $\mathbf{Q}_t$ and $\mathbf{R}_t$ are terminal weight matrices, respectively.

On the other hand, for the constraint of input values, some studies has been conducted [30]. In mentioned research, the bound of these values has been determined using experimental test based on

$$
\begin{aligned}
J = \int_0^d \left[\big(x(t) - x_{ref}\big)^T \mathbf{Q}_p \big(x(t) - x_{ref}\big) + u^T(t) \mathbf{R}_p u(t) \right] dt \\
+ \big(x(d) - x_{ref}\big)^T \mathbf{Q}_t \big(x(d) - x_{ref}\big) + u^T(d) \mathbf{R}_t u(d)
\end{aligned}
\tag{7}
$$

the actual vehicle, so it can be written:

$$-V_{max} \leq V \leq +V_{max}$$
$$-\dot{V}_{max} \leq \Delta V \leq +\dot{V}_{max}$$
$$-\delta_{max} \leq \delta \leq +\delta_{max} \qquad (8)$$
$$-\dot{\delta}_{max} \leq \Delta\delta \leq +\dot{\delta}_{max}$$

Moreover, to avoid collisions with road boundaries, the nonlinear model predictive controller must satisfy the following inequality constraints, where the minimum distance $dist_{min}$ to all obstacles must be greater than a safe distance $dist_{safe}$:

$$dist_{min} \geq dist_{safe} \qquad (9)$$

In this study, the vehicle is modelled as collision box and the distance from vehicle to road curb is computed using check collision in Figure 4.

In general, the automatic turning path tracking control problem is transformed into the following multi-constraint quadratic programming problem:

$$\min\{J(e(t), u(t))\}$$
$$s.t. \begin{cases} -V_{max} \leq V \leq +V_{max} \\ -\dot{V}_{max} \leq \Delta V \leq +\dot{V}_{max} \\ -\delta_{max} \leq \delta \leq +\delta_{max} \\ -\dot{\delta}_{max} \leq \Delta\delta \leq +\dot{\delta}_{max} \\ dist_{min} \geq dist_{safe} \end{cases} \qquad (10)$$

By solving equation (10), one can obtain an optimal control sequence. The initial guesses for the state solutions are defined by straight lines between the initial and target poses of the vehicle.

4. Results and discussion

4.1. Simulation setup

To appraise the performance of the proposed approach in different driving scenarios of both single-point and multi-point turns, simulations are performed in MATLAB software. The parameters of the autonomous vehicle in simulations are provided in Table 2.

Table 2: Values of vehicle model parameters in simulations.

Parameter	Value (unit)
t_w	1.57 m
L_b	2.58 m
W_v	1.77 m
L_v	4.08 m
δ_{max}	33 º
V_{max}	2 m/s
$\dot{\delta}_{max}$	2 º/s
$\dot{V}_{max}$	0.4 m/s²

The inputs of the vehicle including velocity and front steering angle are bounded in the range of [-2, 2] m/s and [-33, 33]o, respectively. Furthermore, the constraints on the rate of the velocity and steering angle are $4T_s$ and $\pi T_s/9$, which $T_s = 0.1$ s. The performance of the proposed method is investigated using the set of the control parameters including the sample time T_s, prediction horizon p, and the control horizon m for the nonlinear model predictive controller and weight matrices for each scenario. Other parameters like safety distance from road boundaries and maximum number of solving iterations are set to be 0.1 m and 40, respectively. In order to improve the stability properties, a sufficiently large prediction horizon must be chosen.

4.2. Validation of the control-oriented model

In this paper, the kinematic model is studied, which is often used for trajectory planning [31]. In this section, the validation of this model for our turn-around task problem is illuminated. To this end, a 9 degrees of freedom (9-DoF) dynamic model is developed which should be compared with kinematic model. The 9-DoF dynamic model is proper to model the longitudinal, lateral, pitch and roll motions. The values of parameters utilized in simulation of the dynamic model are given in Table 3.

Under different conditions and various inputs, modeling errors and limitations of the kinematic model can be verified. The results indicate that errors of kinematic model are ignorable until the velocity of about 7 m/s, which is expected based on known theories [29]. However, turn-around task is a low-speed maneuver, which is consistent with these conditions and is acceptable. Also, the maximum values of input commands in equation (8) is considered in proposed NMPC. Therefore, the similar velocity and steering wheel angle is exerted to these two models.

Table 3: Values of 9-DoF dynamic model parameters.

Parameter	Description	Value (unit)
m	Vehicle mass	2000 kg
l_f	Front overhang	1.23 m
l_r	Rear overhang	1.35 m
L_b	Wheelbase	2.58 m
W_v	Vehicle width	1.77 m
C_d	Drag coefficient	0.3
t_w	Track width	1.57 m
$C_{y,f}$	Front corner stiffness	12 kN/rad
$C_{y,r}$	Rear corner stiffness	11 kN/rad
L_v	Vehicle length	4.08 m
I_{zz}	Yaw polar inertia	4000 kg.m²
C_L	Lift coefficient	0.1

The graphic display of velocity and steering wheel angle commands is depicted in Figure 5-a. The traveled path and yaw rate of kinematic and dynamic model are compared in Figure 5-b and Figure 5-c, respectively. The results show that the kinematic model has a satisfactory performance.

4.3. Single-point turn maneuver

In the following simulation, the control parameters including the output tracking weight matrices $\mathbf{Q}_p$ and $\mathbf{Q}_t$, and the input weight matrices $\mathbf{R}_p$ and $\mathbf{R}_t$ are considered as:

$$\mathbf{Q}_p = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \quad \mathbf{R}_p = 0.01\, I_{2\times2}$$
$$\mathbf{Q}_t = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 100 \end{bmatrix}, \quad \mathbf{R}_t = 0.1\, I_{2\times2} \tag{11}$$
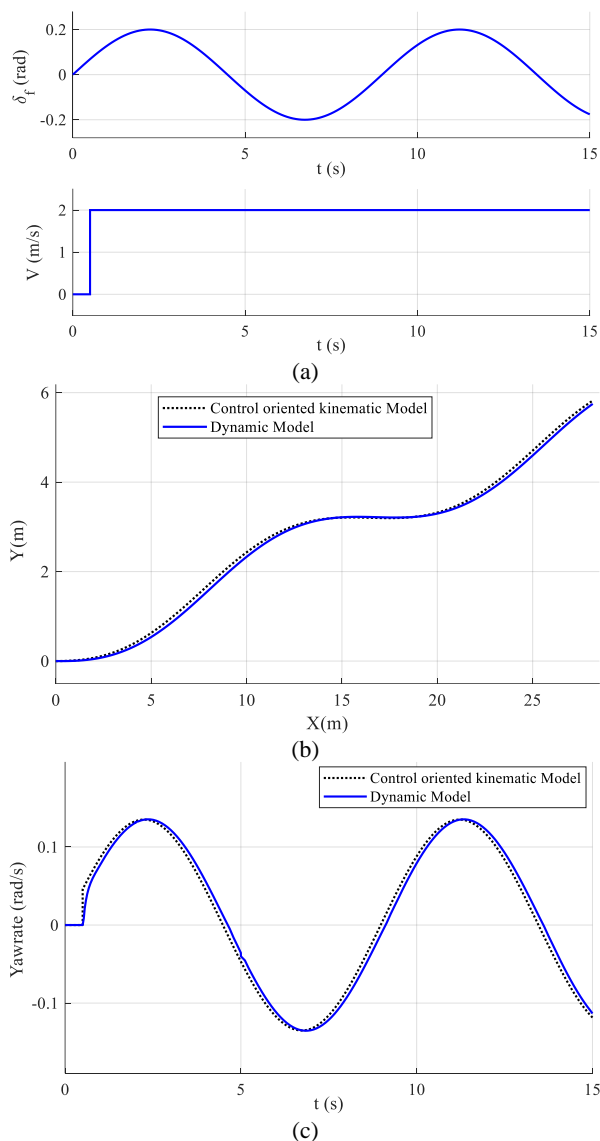

(a)

(b)

(c)

Figure 5: The validation results, (a) Inputs, (b) Travelled path, (c) Yaw rate.

$$\mathbf{Q}_p = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \quad \mathbf{R}_p = 0.01\, I_{2\times2}$$
$$\mathbf{Q}_t = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 100 \end{bmatrix}, \quad \mathbf{R}_t = 0.1\, I_{2\times2} \tag{12}$$

And the prediction and control horizons are selected as 65.

Figure 6 demonstrates the capability of the proposed nonlinear predictive system to generate and follow the real path of the single-point turn. The proposed predictive system is not only able to design the vehicle's path by taking into account the non-collision with the road boundaries, but also is able to follow the designed path. In addition, it is able to set the final values of the lateral position and orientation of the vehicle to the desired values. On the ground that the weight factor corresponding to the final longitudinal position of the vehicle is set to be $[\mathbf{Q}_t]_{1\times1} = 0.1$, the importance of the final longitudinal distance is much less compared to the lateral distance and the final orientation, which is clearly observed in Figure 6.

The time histories of the velocity and front steering angle are presented in Figure 7-a and -b, respectively. Both of the control inputs lie in the prescribed allowable regions. Practically speaking, after an initial assessment of the scene by an expert driver in order to trade-off between the required traveled longitudinal and lateral distances, he tries to make use of the full capability of both velocity and front steering angle of the vehicle, and when time passes, he decreases the steering angle. The results of the velocity and front steering angle in Figure 7 justifies the expected behavior for the autonomous vehicle.

4.4. Multi-point turn maneuver

4.4.1. Case one: Three-point turn

In this maneuver, the control parameters including the weighting matrices $\mathbf{Q}_p$ and $\mathbf{Q}_t$, and
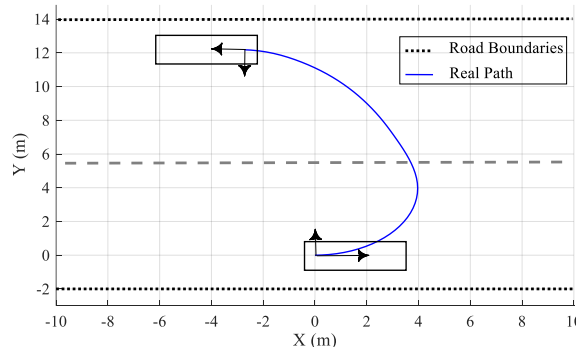

Figure 6: The real path in single-point turn task.

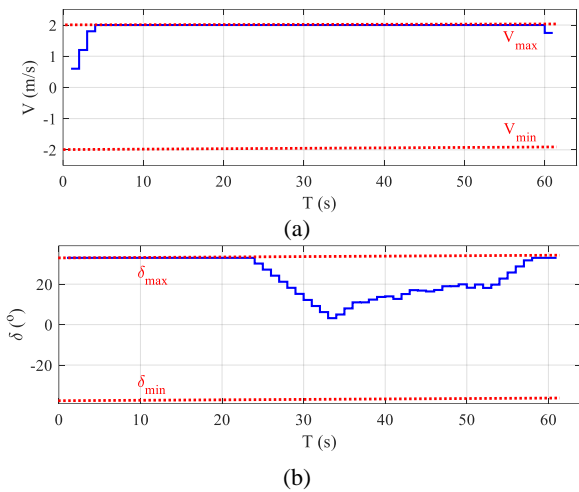**Figure 7:** The control inputs in single-point turn task. (a) Velocity, (b) Front steering angle.

the input weight matrices $\mathbf{R}_p$ and $\mathbf{R}_t$ are considered as below.

$$\mathbf{Q}_p = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \quad \mathbf{R}_p = 0.01\, I_{2\times2}$$
$$\mathbf{Q}_t = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 100 \end{bmatrix}, \quad \mathbf{R}_t = 0.1\, I_{2\times2} \tag{13}$$

And the prediction horizon and control horizon are selected as 100 and 70, respectively.

Figure 8 shows the ability of proposed strategy for the three-point turn. In operating tactics for three-point turn, the car movement is started by turning forward from a specific position and then the heading direction of the car is adjusted by two times changing back and forth in the allowed space. Finally, the remaining section of the maneuver is done by turning forward without exceeding the opposite border of the road.

The velocity and steering angle of the three-point turn are shown in   Figure 9-a and -b, respectively.
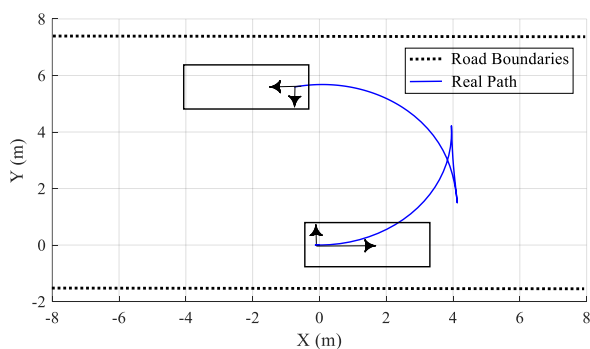

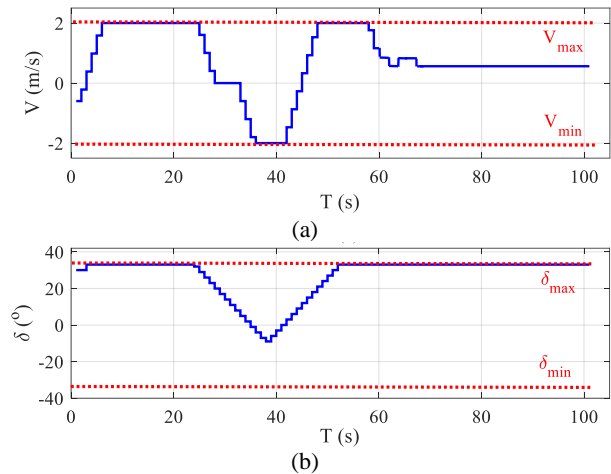**Figure 8:** The real path in three-point turn task.

Figure 9: The control inputs in three-point turn task. (a) Velocity, (b) Front steering angle.

The amounts of the control inputs are bounded in the allowed range. As depicted in   Figure 9, the proposed nonlinear predictive method evaluates the geometry of the road and analyses the collision avoidance constraints, then predicts the minimum number of command to accomplish the task. The advantage of this predictive path selection is that the NMPC prevents the excess rotation of the vehicle in the middle of turning, which subsequently has decreased the steering angle and reduced tire wearing with the ground.

4.4.2. Case two: Five-point turn

In order to demonstrate the capability of the proposed approach in accomplishing turn-around task on narrower roads, five-point turn maneuver is also investigated. In the simulation of this maneuver, the control parameters including $\mathbf{Q}_p$ and $\mathbf{Q}_t$, and the input weight matrices $\mathbf{R}_p$ and $\mathbf{R}_t$ are considered as:

$$\mathbf{Q}_p = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{R}_p = 0.01\, I_{2\times2}$$
$$\mathbf{Q}_t = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 100 \end{bmatrix}, \quad \mathbf{R}_t = 0.1\, I_{2\times2} \tag{14}$$

And the prediction horizon and control horizon are selected as 95 and 115, respectively.

Figure 10 and Figure 11 reveal the performance of the assistance system subject to the relevant restrictions and constraints.
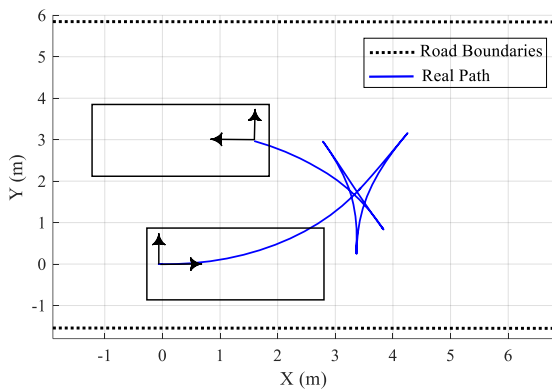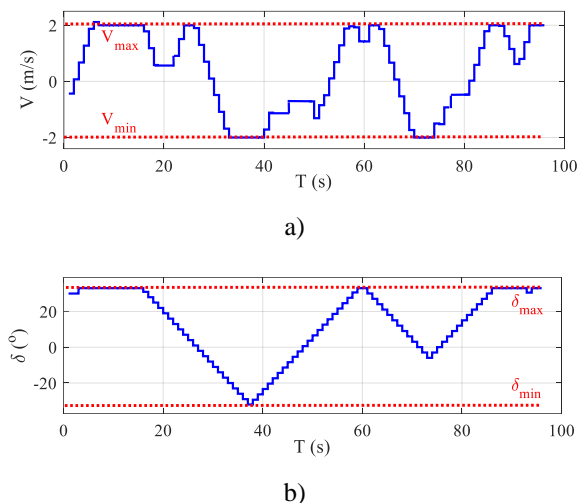
Figure 10: The real path in Five-point turn task.

a)

b)

Figure 11: The control inputs in five-point turn task. (a) Velocity, (b) Front steering angle.

5. Conclusions

A new framework for simultaneous planning and control of AVs in turn-around tasks is proposed. To this end, an approach is proposed based on nonlinear model predictive control which has sufficient accuracy for this problem rather than linear one. The constraints of speed and steering angle as well as collision avoidance with the road side are considered in the simulations. The results confirmed the excellent performance of the chosen approach in the motion planning and control process of turn around task. With this system, we achieve a higher turning precision, larger safety margin, and less turning time in turn around maneuver. As future works, we are studying the problem of decision-making and planning of turn around task in the presence of other vehicles and road users.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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