



Design of an Adaptive Fuzzy Controller for Antilock Brake Systems

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ABSTRACT

The control of Antilock Braking Systems (ABS) is a difficult problem, because of their nonlinearities and uncertainties appearing in their dynamics and parameters. To overcome these issues, this paper proposes a new adaptive controller for the next generation of ABS. After considering a complex vehicle dynamic, a triple adaptive fuzzy control system is presented. Important parameters of the vehicle dynamic include two separated brake torques for front and rear wheels, as well as longitudinal weight transfer which is caused by the acceleration or deceleration. Because of the nonlinearity of the vehicle dynamic model, three fuzzy-estimators have been suggested to eliminate nonlinear terms of the front and rear wheels' dynamic. Since the vehicle model parameters change due to variations of road conditions, an adaptive law of the controller is derived based on Lyapunov theory to adapt the fuzzy-estimators and stabilize the entire system. The performance of the proposed controller is evaluated by some simulations on the ABS system. The results demonstrate the effectiveness of the proposed method for ABS under different road conditions.

1. Introduction

Automotive antilock braking systems are designed to stop vehicles by preventing wheel lock up and reducing the stopping distance. These two performances are achieved by maintaining lateral stability due to the increased high lateral friction coefficient. Designing a controller of ABSs is not a facile method regarding nonlinearity of vehicle model dynamics, complicated operation of the controller at unstable equilibrium point and variations of model parameters over a wide range which are caused by variations of road and vehicle conditions [1]. Hence a good pragmatic controller needs to be adaptable and robust enough to provide sufficient security for passengers [2]. Early research into ABS were developed in 1930s for aircraft and conducted by Bosch after that Teldix GmbH, constructed first

generation of ABS systems that stopped wheels without lock-up. Second generation of ABS was used in a series of S-class Mercedes in 1978 [3].

ABS is applicable approximately to all kinds of vehicle and can be successfully integrated into air and hydraulic brake systems. In former systems, ABS consists of a hydraulic modulator (control valve), control electronics (central unit), and sensors mounted on wheels [4]. Sensors constantly measure braking power and when braking is so intensive that may block wheels, controller in central electronic unit sends the command to hydraulic system to open the electromagnetic valves. This reduces the pressure of oil in hydraulic system, and consequently the forces on the brake disks [5].

Conventional and commercial ABSs use a look-up table to compute the desired braking torque value using the velocity and acceleration data of

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the vehicle and wheels [6]. Recently, some companies have applied separate sensing systems to determine vehicle velocity, rather than estimating it from the wheel data [7].

Numerous control schemes are reported in the literature for controlling the ABSs, such as variable structure control [8], adaptive control [9], predictive control [10], neural control [11], fuzzy control [12], fuzzy sliding mode control [13, 14], adaptive backstepping control [15], output feedback control [16], dynamic programming [6] and PID control [17]. Each approach has its own advantages and disadvantages. The major drawback of sliding mode control is sensitivity to the uncertainty caused by system variations with respect to the nominal model. This phenomenon can excite the unmodeled states or parameter variations and makes the system unstable. PID controller is one of the most common controllers due to its simple structure and easy to implement. However, it does not possess enough robustness in the presence of model uncertainties and nonlinear dynamics. For solving this problem, Ref. [17] applied a new Nonlinear PID controller to a class of truck ABS problems. This controller combines the advantages of robust control and easy tuning. Simulation results at various situations show that controller has shorter stopping distance and better performance than the conventional PID controller. In [18] an nonlinear control method based on time-varying Asymmetric Barrier Lyapunov Function is proposed to track the optimal slip ratio and guarantee violation on the slip ratio constraints. However, this method requires many road tests and cannot make full use of the road adhesion capability.

An ABS control algorithm based on nonlinear backstepping design schemes is designed in [15]. Results show that the method can obtain the rapid dynamic responses of the fixed control model, however, it does not achieve the convergence of the adaptive control model.

Ref. [19] proposed a PSO based fuzzy neural network controller for ABS System in electric vehicle. The results show that integrated control of ABS maintains the safe distance from obstacle without sacrificing the performance of either system. To improve the robustness of brake controller and inhibit the uncertainty of parameter of ABS, a feed-forward controller using the flatness-based inverse was proposed in [20], however, its domain is restricted. A self-learning fuzzy sliding mode controller is proposed in [1] for ABS. Fuzzy controller is the main tracking controller, which is used to mimic an ideal controller; and a robust controller is derived to

compensate for the difference between the ideal controller and the fuzzy controller.

In [21] a fuzzy variable structure control method using fuzzy control is designed for is applied to antilock braking system. The results show that the fuzzy variable structure controller can reduce chattering and fast response of conventional variable structure control. Ref. [22] designed a hybrid control system using a recurrent neural network observer for antilock brake systems, which functions as a means to estimate the uncertainty of the system to reduce chattering phenomena caused by sliding mode control. Also [23] also introduced a self-learning fuzzy control system integrated with sliding mode control to reduce the dependency of the controller on vehicle model by modulating the brake torque for optimum braking with enhanced reduction in chattering. More contribution in using different methods prevails thanks to the works of a number of scholars [24-27].

Many different control techniques have been introduced to improve the performance and efficiency of ABS. Most of these approaches require system models, and some of them cannot achieve satisfactory performance under the changes of various road conditions. Due to the limitations of the execution speed of the hydraulic actuator and transmission delay of the hydraulic braking lines for conventional hydraulic braking systems, most practical products still use logic control based on experience [28].

Fuzzy logic and fuzzy sliding mode based controllers are both effective methods and capable of presenting acceptable systems. However, their major drawback is that the fuzzy rules should be previously tuned by time-consuming trial-and-error procedures. To overcome this drawback, in this paper an adaptive fuzzy control based on the Lyapunov synthesis approach is designed for the ABS. With this approach, the fuzzy rules can be automatically adjusted to achieve satisfactory system response by an adaptive law. Another significant feature of these types of controllers is that it has superior responses without chattering phenomenon in control inputs compared to other controllers. In details, a sophisticated real-world prototype is used, which takes the longitudinal weight transfer caused by the acceleration or deceleration into account. An adaptive fuzzy controller is applied to deal with this system controlling the rear and front braking torques separately using three nonlinear fuzzy-estimators that increase the capabilities of the proposed ABS control system compared to the previous systems. The paper is organized as follows. Section 2 introduces a dynamic model of the vehicle, which

is used for designing and simulating the controller. The friction model is employed to ensure the stability of the proposed controller for various road conditions. Section 3 presents a fuzzy system as a universal approximator. Section 4 explains the suggested adaptive control system and proves its stability, and finally section 5 contains information about simulation results.

2. Vehicle Dynamic and Tire Friction Models

The most essential goal in designing a controller for ABSs is to robustly regulate the real wheel slip at its optimum value and in the shortest time. In this paper, the vehicle model given in [29] has been used as the dynamic model to design the control system. Figure 1 shows the vehicle free body diagram.

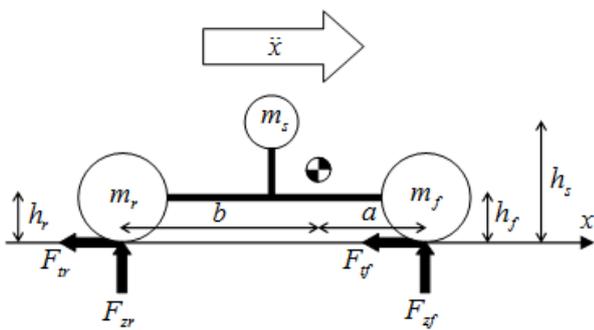


Fig. 1: The vehicle free body diagram

The tractive forces produced by the tire are proportional to the normal forces acting on the tire, which is called the road coefficient of adhesion, (μ =friction coefficient) which varies depending on road conditions. Research shows that the friction coefficient is a nonlinear function of wheel slip (λ) in a specified road condition (Fig. 2).

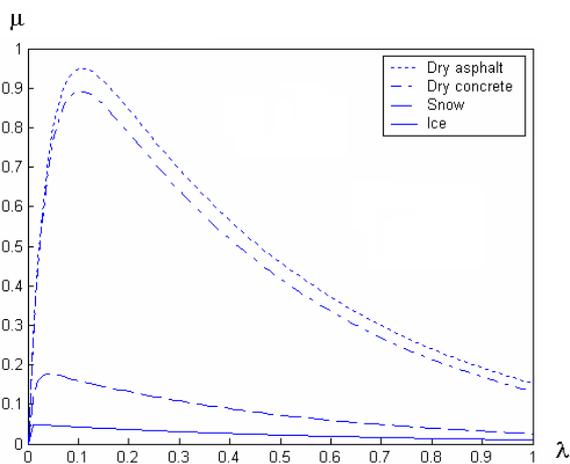


Fig. 2: Atypical $\mu - \lambda$ curve

Assuming $x, v, \omega_f, \omega_r, T_{bf}$ and T_{br} as the vehicle position, vehicle speed, angular velocity of the front and rear wheel, front and rear brake torque, respectively, we have:

$$\begin{cases} \dot{x} = v \\ \dot{v} = -g \frac{\mu(\lambda_r)m_1 + \mu(\lambda_r)m_2}{m_{tot} - \mu(\lambda_f)m_3 + \mu(\lambda_r)m_3} \\ \dot{\omega}_f = \frac{1}{2J_f} (-T_{bf} + \mu(\lambda_f)m_1R_{\omega}g - \mu(\lambda_f)m_3R_{\omega}\ddot{x}) \\ \dot{\omega}_r = \frac{1}{2J_r} (-T_{br} + \mu(\lambda_r)m_2R_{\omega}g + \mu(\lambda_f)m_3R_{\omega}\ddot{x}) \end{cases} \quad (1)$$

where

$$m_1 = \frac{b}{a+b}m_{tot}, m_2 = \frac{a}{a+b}m_{tot}, m_3 = \frac{m_f h_f + m_s h_s + m_r h_r}{a+b}$$

Table 1 defines the constants and their nominal values used in equation (1). Since the main goal is to control the wheel slip, the state space equations should be rewritten based upon the front and rear wheel slips.

Table 1: Vehicle Model Data [29]

Symbol	Quantity	Value
g	Acceleration due to gravity	9.81 m/s ²
a	Distance from center of gravity to front axle	1.186 m
b	Distance from center of gravity to rear axle	1.258 m
h_s	Height of the sprung mass	0.6 m
h_f	Height of front unsprung mass	0.3 m
h_r	Height of rear unsprung mass	0.3 m
m_{tot}	Total mass of the vehicle	1500 kg
m_s	Sprung mass of the vehicle	1285 kg
m_f	Front unsprung mass	96 kg
m_r	Rear unsprung mass	119 kg
J_f	Moment of inertia of the front wheel	1.7 kg.m ²
J_r	Moment of inertia of the rear wheel	1.7 kg.m ²
R_{ω}	Radius of tire	0.326 m

The front and rear wheel slip and their derivatives are defined as:

$$\lambda_f = \frac{v - \omega_f R_{\omega}}{v} \Rightarrow \dot{\lambda}_f = \frac{v(1 - \lambda_f) - \dot{\omega}_f R_{\omega}}{v} \quad (2)$$

$$\lambda_r = \frac{v - \omega_r R_{\omega}}{v} \Rightarrow \dot{\lambda}_r = \frac{v(1 - \lambda_r) - \dot{\omega}_r R_{\omega}}{v} \quad (3)$$

Substituting (2) and (3) into (1), we obtain:

$$\begin{cases} \dot{x} = v \\ \dot{v} = f_1(\lambda_f, \lambda_r) \\ \dot{\lambda}_f = \frac{f_1(\lambda_f, \lambda_r)(1 - \lambda_r) - f_2(\lambda_f, \lambda_r) + u_f}{v} \\ \dot{\lambda}_r = \frac{f_1(\lambda_f, \lambda_r)(1 - \lambda_r) - f_3(\lambda_f, \lambda_r) + u_r}{v} \end{cases} \quad (4)$$

where,

$$f_1(\lambda_f, \lambda_r) = -g \frac{\mu(\lambda_f)m_1 + \mu(\lambda_r)m_2}{m_{tot} - \mu(\lambda_f)m_3 + \mu(\lambda_r)m_3} \quad (5)$$

$$f_2(\lambda_f, \lambda_r) = \frac{R_\omega}{2J_f} (\mu(\lambda_f)m_1R_\omega g - \mu(\lambda_f)m_3R_\omega f_1) \quad (6)$$

$$f_3(\lambda_f, \lambda_r) = \frac{R_\omega}{2J_r} (\mu(\lambda_r)m_2R_\omega g + \mu(\lambda_r)m_3R_\omega f_1) \quad (7)$$

$$u_f = \frac{R_\omega T_{bf}}{2J_f} \quad (8)$$

$$u_r = \frac{R_\omega T_{br}}{2J_r} \quad (9)$$

In this paper, the tire friction model introduced by Burckhardt [30] has been used to simulate the performance of the presented antilock brake system. It provides the tire-road coefficient of friction, μ , as a function of the wheel slip, λ , and the vehicle velocity, v . Table 2 shows the model parameters for various road conditions.

$$\mu_x(\lambda, v) = (C_1(1 - e^{-C_2\lambda}) - C_3\lambda)e^{-C_4\dot{\lambda}} \quad (10)$$

where C_4 is in the range of 0.02-0.04 s/m.

Table 2: Friction Model Parameters [31]

Surface conditions	c_1	c_2	c_3
Dry asphalt	1.2801	23.99	0.52
Wet asphalt	0.857	33.822	0.347
Dry concrete	1.1973	25.168	0.5373
Snow	0.1946	94.129	0.0646
Ice	0.05	306.39	0

3. Fuzzy System as a Universal Approximator

A multiple-input single-output (MISO) fuzzy system is a nonlinear mapping from an input vector to an output. To define a MIMO fuzzy system with m outputs, simply define m MISO fuzzy systems. The fuzzy system is characterized by a set of p if-then rules, stored in a rule-base, expressed as [12]:

$$\begin{aligned} R_1 : & \text{ If } (x_1 \text{ is } F_1^1 \text{ and } \dots \text{ and } x_n \text{ is } F_n^1) \text{ then } f = c_1 \\ & \vdots \\ R_p : & \text{ If } (x_1 \text{ is } F_1^p \text{ and } \dots \text{ and } x_n \text{ is } F_n^p) \text{ then } f = c_p \end{aligned}$$

where F_j^i denoted the i^{th} membership function

associated with j^{th} input variable. A two-input fuzzy system with a triangular membership function can be expressed as:

$$f(x_1, x_2, \theta) = \theta^T \zeta(x_1, x_2) \quad (11)$$

where, $\theta = [c_1, \dots, c_p]^T$ and

$\zeta(x_1, x_2) = [\zeta_1, \dots, \zeta_p]^T$. The parameter ζ_i can be obtained by the center average defuzzification as as the following [32]:

$$\zeta_i = \frac{\min(\mu_{F_1^i}(x_1), \mu_{F_2^i}(x_2))}{\sum_{j=1}^p \min(\mu_{F_1^j}(x_1), \mu_{F_2^j}(x_2))} \quad (12)$$

In the above equation, $\mu_{F_1^j}(x_1), \mu_{F_2^j}(x_2)$ are membership degrees of x_1 and x_2 in membership functions F_1^j and F_2^j respectively. Fuzzy system (11) with triangular membership function and center average defuzzification (Eq. 12) is a universal approximator on an interval $R = [a, b]$ [33]. Therefore, an ideal fuzzy function can be found that approximates any function $f(x_1, x_2)$.

This function is illustrated as:

$$f(x_1, x_2) = f(x_1, x_2, \theta^*) + e \quad (13)$$

where $|e| \leq E$.

4. Adaptive Fuzzy Controller

According to the universal approximation theorem, the following series of ideal fuzzy functions are proposed:

$$f_i(\lambda_f, \lambda_r) = f_i^*(\lambda_f, \lambda_r, \theta_i^*) + e_i, i = 1, 2, 3 \quad (14)$$

where,

$$f_i^*(\lambda_f, \lambda_r, \theta_i^*) = \theta_i^{*T} \zeta_i(\lambda_f, \lambda_r) \quad i = 1, 2, 3 \quad (15)$$

Assuming an estimation for the above fuzzy functions as,

$$\hat{f}_i(\lambda_f, \lambda_r, \hat{\theta}_i) = \hat{\theta}_i^T \zeta_i(\lambda_f, \lambda_r), i = 1, 2, 3 \quad (16)$$

The goal of the adaptive fuzzy controller is to find an appropriate adaptive law and control the inputs to minimize the errors $\tilde{\theta}_i, e_f, e_r$ which are defined as:

$$\tilde{\theta}_i = \hat{\theta}_i - \theta_i^* \quad i = 1, 2, 3 \quad (17)$$

$$e_f = \lambda_f - \lambda_{fd} \quad (18)$$

$$e_r = \lambda_r - \lambda_{rd} \quad (19)$$

Theorem 1: The ideal model, (4) with the following control inputs (Eq. 20) and adaptive laws (Eq. 21), is stable.

$$u_f = \hat{f}_2 - (1 - \lambda_f) \hat{f}_1 + v \left(\dot{\lambda}_{fd} - k e_f \right) - \frac{e_f \left((1 - \lambda_f)^2 + 1 \right) + e_r (1 - \lambda_f) (1 - \lambda_r)}{v} \quad (20)$$

$$u_r = \hat{f}_3 - (1 - \lambda_r) \hat{f}_1 + v \left(\dot{\lambda}_{rd} - k e_r \right) - \frac{e_r \left((1 - \lambda_r)^2 + 1 \right) + e_f (1 - \lambda_f) (1 - \lambda_r)}{v}$$

$$\dot{\hat{\theta}}_1 = \frac{1}{\eta_1} \left(\frac{e_f (1 - \lambda_f) + e_r (1 - \lambda_r)}{v} \zeta_1 (\lambda_f, \lambda_r) - \sigma \hat{\theta}_1 \right)$$

$$\dot{\hat{\theta}}_2 = - \frac{1}{\eta_2} \left(\frac{e_f}{v} \zeta_2 (\lambda_f, \lambda_r) + \sigma \hat{\theta}_2 \right) \quad (21)$$

$$\dot{\hat{\theta}}_3 = - \frac{1}{\eta_3} \left(\frac{e_r}{v} \zeta_3 (\lambda_f, \lambda_r) + \sigma \hat{\theta}_3 \right)$$

Proof: Assuming a Lyapunov function,

$$V = \frac{1}{2} e_f^2 + \frac{1}{2} e_r^2 + \frac{1}{2} \eta_1 \tilde{\theta}_1^T \tilde{\theta}_1 + \frac{1}{2} \eta_2 \tilde{\theta}_2^T \tilde{\theta}_2 + \frac{1}{2} \eta_3 \tilde{\theta}_3^T \tilde{\theta}_3 \quad (22)$$

Differentiating (Eq. 22) with respect to time and using (Eq. 17), we have:

$$\dot{V} = e_f \dot{e}_f + e_r \dot{e}_r + \eta_1 \tilde{\theta}_1^T \dot{\tilde{\theta}}_1 + \eta_2 \tilde{\theta}_2^T \dot{\tilde{\theta}}_2 + \eta_3 \tilde{\theta}_3^T \dot{\tilde{\theta}}_3 \quad (23)$$

Using Eq. (4), (18), (19) and (20), it can be shown that:

$$\dot{e}_f = \frac{(f_1 - \hat{f}_1)(1 - \lambda_f) - (f_2 - \hat{f}_2)}{v} - k e_f - \frac{e_f \left((1 - \lambda_f)^2 + 1 \right) + e_r (1 - \lambda_f) (1 - \lambda_r)}{v^2} \quad (24)$$

$$\dot{e}_r = \frac{(f_1 - \hat{f}_1)(1 - \lambda_r) - (f_3 - \hat{f}_3)}{v} - k e_r - \frac{e_r \left((1 - \lambda_r)^2 + 1 \right) + e_f (1 - \lambda_f) (1 - \lambda_r)}{v^2} \quad (25)$$

Using Eq. (17), it can be defined that:

$$\tilde{f}_i (\lambda_f, \lambda_r, \hat{\theta}_i) = \tilde{\theta}_i^T \zeta_i (\lambda_f, \lambda_r)$$

$$= \hat{f}_i (\lambda_f, \lambda_r, \hat{\theta}_i) - f_i^* (\lambda_f, \lambda_r, \hat{\theta}_i), i = 1, 2, 3$$

Substituting Eq. (21), (24) and (25) into (23), it is shown that:

$$\dot{V} = e_f \frac{e_1 (1 - \lambda_f) - e_2}{v} - e_f \frac{\tilde{f}_1 (1 - \lambda_r) - \tilde{f}_2}{v} - k e_f^2 - \frac{e_f^2 \left((1 - \lambda_f)^2 + 1 \right) + e_r e_f (1 - \lambda_f) (1 - \lambda_r)}{v^2}$$

$$+ e_r \frac{e_1 (1 - \lambda_r) - e_3}{v} - e_r \frac{\tilde{f}_1 (1 - \lambda_r) - \tilde{f}_3}{v} - k e_r^2 - \frac{e_r^2 \left((1 - \lambda_r)^2 + 1 \right) + e_f e_r (1 - \lambda_f) (1 - \lambda_r)}{v^2}$$

$$+ \tilde{\theta}_1^T \frac{e_f (1 - \lambda_f) + e_r (1 - \lambda_r)}{v} \zeta_1 (\lambda_f, \lambda_r)$$

$$- \tilde{\theta}_2^T \frac{e_f}{v} \zeta_2 (\lambda_f, \lambda_r) - \tilde{\theta}_3^T \frac{e_r}{v} \zeta_3 (\lambda_f, \lambda_r)$$

$$- \sigma \tilde{\theta}_1^T \hat{\theta}_1 - \sigma \tilde{\theta}_2^T \hat{\theta}_2 - \sigma \tilde{\theta}_3^T \hat{\theta}_3$$

Simplifying Eq. (26), we obtain:

$$\dot{V} = e_1 \frac{e_f (1 - \lambda_f) + e_r (1 - \lambda_r)}{v} - e_2 \frac{e_f}{v} - e_3 \frac{e_r}{v} - k e_f^2 - k e_r^2 - \frac{(e_f (1 - \lambda_f) + e_r (1 - \lambda_r))^2}{v^2}$$

$$- \frac{e_f^2}{v^2} - \frac{e_r^2}{v^2} - \sigma \tilde{\theta}_1^T \hat{\theta}_1 - \sigma \tilde{\theta}_2^T \hat{\theta}_2 - \sigma \tilde{\theta}_3^T \hat{\theta}_3 \quad (27)$$

Using the following inequality, $-x^2 \pm 2xy \leq y^2$, we have:

$$e_1 \frac{e_f (1 - \lambda_f) + e_r (1 - \lambda_r)}{v} - \frac{(e_f (1 - \lambda_f) + e_r (1 - \lambda_r))^2}{v^2} \leq \frac{e_1^2}{2}$$

$$- \frac{e_f^2}{v^2} - e_2 \frac{e_f}{v} \leq \frac{e_2^2}{2} \quad (28)$$

$$- \frac{e_r^2}{v^2} - e_3 \frac{e_r}{v} \leq \frac{e_3^2}{2} \quad (28)$$

And using the following inequality,

$$-x^T x \pm x^T y \leq \frac{-x^T x}{2} + \frac{y^T y}{2}, \text{ we have:}$$

$$- \tilde{\theta}_i^T \hat{\theta}_i = - \tilde{\theta}_i^T (\tilde{\theta}_i + \theta_i^*) \leq - \frac{|\tilde{\theta}_i|^2}{2} + \frac{|\theta_i^*|^2}{2} \quad (29)$$

$$i = 1, 2, 3$$

Rewriting (27) by exploiting (28) and (29), following inequality is obtained:

$$\dot{V} \leq -k (e_f^2 + e_r^2) - \frac{\sigma}{2} \left(|\tilde{\theta}_1|^2 + |\tilde{\theta}_2|^2 + |\tilde{\theta}_3|^2 \right) + d \quad (30)$$

where,

$$d = \frac{E_1^2 + E_2^2 + E_3^2 + \sigma |\theta_1^*|^2 + \sigma |\theta_2^*|^2 + \sigma |\theta_3^*|^2}{2}$$

$$|e_i| \leq E_i \quad i = 1, 2, 3$$

By defining b_e and $b_{\tilde{\theta}}$ respectively as a circle of radius $\sqrt{d/k}$, and a sphere of radius $\sqrt{2d/\sigma}$, and using theorem of appendix A, it can be shown that the error is bounded and the system stability is assured.

5. Simulation Results

Equations (20) and (21) show that the main controller consists of three fuzzy functions. These functions have two inputs that are front and rear wheel slip, respectively. Figure 3 indicates the considered input membership functions of the fuzzy functions. (26)

In order to have high accuracy in the proposed system, the upper limit of the wheel slips in membership functions was considered 0.2; because the value of wheel slips do not exceed 0.2 during braking operations.

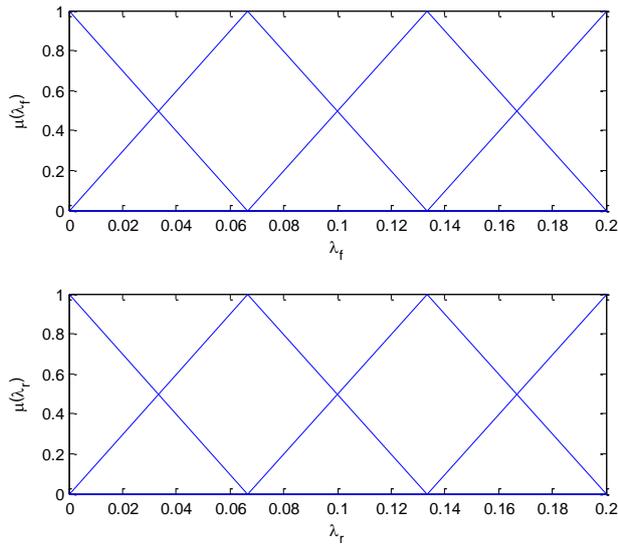


Figure 3: Input membership functions of the proposed front and rear fuzzy controllers

To evaluate the performance of the proposed controller, a braking scenario consisting of three different phases has been considered. First, it has been supposed that a vehicle, the specifications of which explained in table 1, starts braking on the asphalt road with the initial velocity of 20 m/s (equal to 72 km/hr). It has been also supposed that after passing 5 m during the braking process, the condition of the road changes to wet asphalt. In the last phase, after 15 m, the road condition changes to snowy road. During these phases, the main objective is to control the rear and front wheel slips on the set point with value of 0.15. A remained point is that with considering of the equation (4), the slippage of the wheels cannot be determined when the vehicle velocity approaches zero.

To pave the way, it has been supposed that the simulation of the braking process continues as long as the velocity reaches 1 m/s. The braking system with above-mentioned specifications has been simulated using MATLAB Simulink as depicted in figure 4.

Wheel slip, its error, and brake torque of rear and front wheels have been shown in figures 5, 6, and 7, respectively. Except on the points in which the road conditions change, slippage error is tremendously small in all other points, as shown in figure 6. The result shows that the adaptability of the fuzzy estimators to compensate the changes of the model is high. It has been demonstrated that suitable brake torque control inputs have been obtained, as it can be seen in figure 7, that are convincing enough to implement for industrial applications. For instance, the maximum value of

the control inputs might be applicable to industrial brake systems.

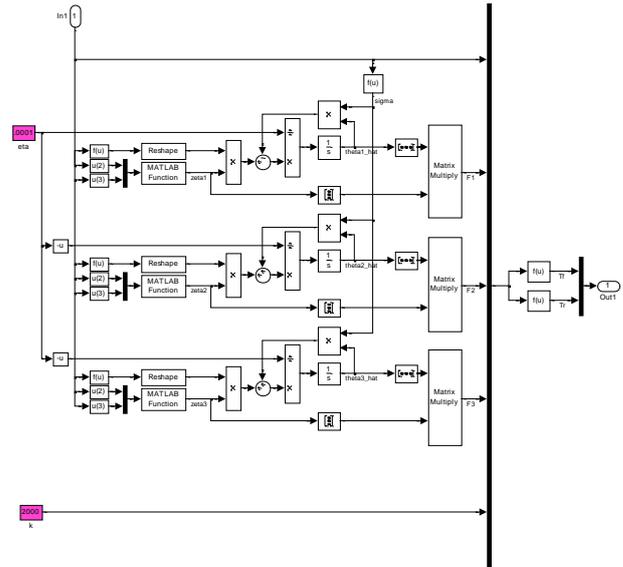


Figure 4: The designed controller in MATLAB Simulink

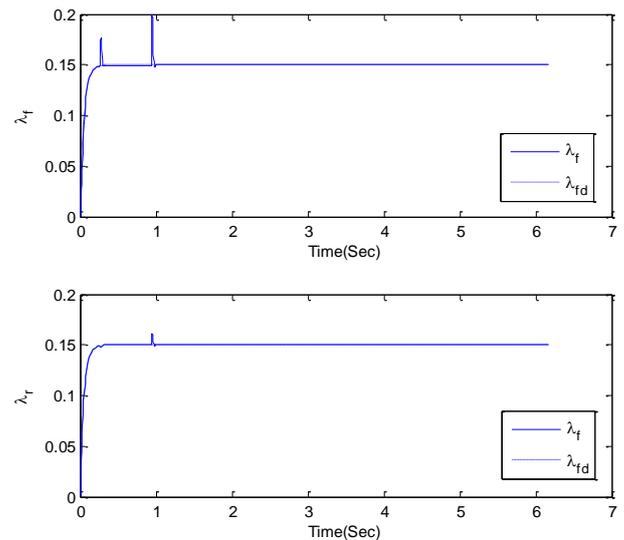


Figure 5: the slippage of rear and front wheels during braking process

6. Conclusion

A novel adaptive controller based on fuzzy estimators has been presented for ABSs. First, a four-wheel model including separate brake torques of rear and front wheels was considered. Secondly, three fuzzy estimators were considered to eliminate the nonlinear terms of the model. Subsequently, to adapt the fuzzy-estimators and stabilize the system, an adaptive control law based upon Lyapunov theorem was proposed. Thirdly, a three-step braking scenario was considered in order to evaluate the performance of the proposed

controller. Finally, the simulation results demonstrated that the system is able to prevent wheels from slipping more than 0.15 in road conditions.

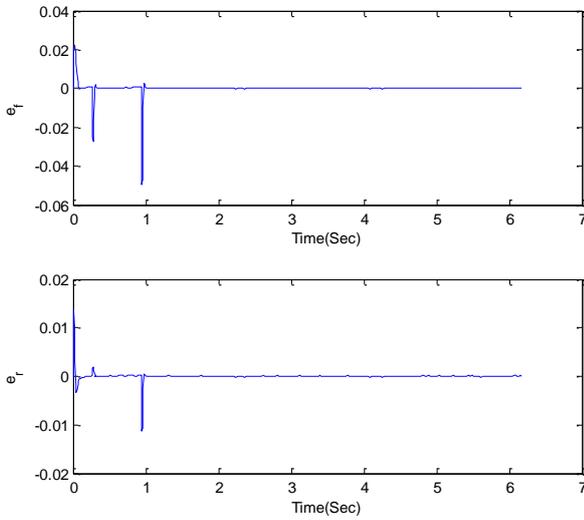


Figure 6: the slippage error of rear and front wheels during braking process

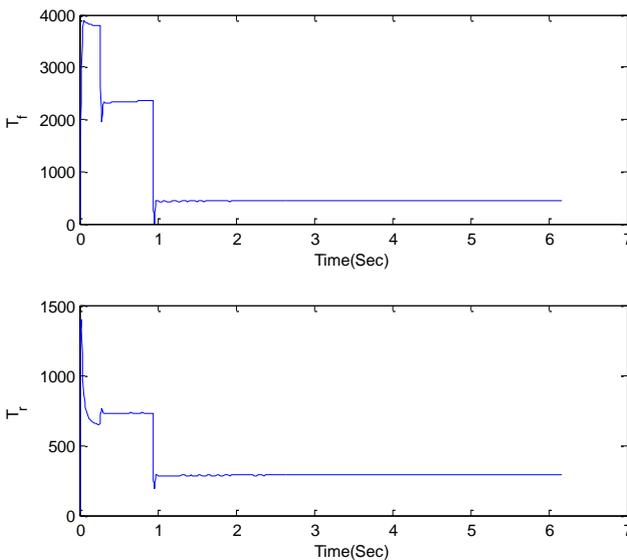


Figure 7: the brake torques of rear and front wheels during braking process

Appendix A

Theorem [33]: Let $V : \mathbb{R}^q \times \mathbb{R}^p \rightarrow \mathbb{R}$ be a continuously differentiable function such that:

$$\gamma_{e1}(|e|) + \gamma_{\tilde{\theta}1}(|\tilde{\theta}|) \leq V(e, \tilde{\theta}) \leq \gamma_{e2}(|e|) + \gamma_{\tilde{\theta}2}(|\tilde{\theta}|)$$

where $\gamma_{e1}, \gamma_{e2}, \gamma_{\tilde{\theta}1}, \gamma_{\tilde{\theta}2}$ are class- K_∞ . Assume that for a given error system, a control law is defined such that both $|e| \geq b_e$ and $|\tilde{\theta}| \geq b_{\tilde{\theta}}$ implies $\dot{V} \leq 0$. Then, $e \in B_e$ for all t with:

$$e \in B_e = \{e \in \mathbb{R}^p : |e| \leq \gamma_{e1}^{-1}(\max(V(0), V_r))\}$$

where,

$$V_r = \gamma_{e2}(b_e) + \gamma_{\tilde{\theta}2}(b_{\tilde{\theta}}).$$

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