A new approach for Modeling and Evaluation of efficiency and power generation in Sterling engine; Analytical study

H.Saberinejad[a], A.Keshavarz[b*] ,M.Bastami[c] and M.Payandehdoost[a]

a PhD Student, Faculty of mechanical engineering, K.N. Toosi university of Technology, b Professor , Faculty of mechanical engineering, K.N. Toosi university of Technology, c Master science , Faculty of mechanical engineering, K.N. Toosi university of Technology,

Keshavarz@kntu.ac.ir

Abstract

Although, the Stirling engine (SE) was invented many years ago, the investigation on SE is still interesting due to variety of energy resources can be applied to power it (solar energy, fossil fuel, biomass and geothermal energy). In this paper, the thermodynamic cycle of SE is analyzed by employing a new analytical model and a new method is presented to evaluate output power and efficiency of real engines. Using the correcting functions; represent more accurate results for known Schmidt equations respect to adiabatic model. So without need to employing numerical methods and iterative solver programs, analogous results with accuracy and correctness of open-form solution-adiabatic method is obtained. The modeling of results of two methods is done by Non-linear Multiple Regression and new equations based on Schmidt equations with new correctness factors is presented. The correctness factors are function of structural and operational characteristics of engine. Moreover, available output data of GPU-3 SE was compared. These comparisons show good agreement, indicating that the model is an appropriate method for modeling of SE outputs.

Keywords: *Stirling engine, analytical solution, non-linear regression, thermodynamic analysis*

1. Introduction

Using of SE is one of the most important green-energy technologies. Due to many advantages, this old invention has fascinated much attention of many researchers in industrial and academic centers recently [1-4]. Some of the these advantages are: low noise, high thermal efficiency, requiring little maintenance, low level of toxic emission if used by fuel and being pliable to applied almost any kind of heat source. Despite some disadvantages existed in SEs; they are successfully applied in the micro combined heat and power (CHP) system and power generation currently. Inability to rapid changing power, lower power to weight ratio and exclusive technology has prevented to develop SEs to more general fields. In the event that these problems are solved by new researches, this old invention might become more common in future.

Generally, the thermodynamic analysis of SE can be classified to open-form (implicit equations) and close-form solutions (explicit equations).

In closed-form, the solution of governing equations is obtained explicitly and no iterative numerical methods are necessary. The Schmidt solution is the most prominent closed-form solution to analyze SE [5]. Considerable efforts have been made to develop and improve the Schmidt analysis considering engine losses [6-8].

The adiabatic analysis is an open-form method. In this method, the whole work spaces of engine are considered adiabatic and the heat transfer occurs only in cold and hot heat exchangers. The working fluid leaves the cold and hot heat exchangers exactly in their temperature respectively. In such methods the heat regenerating process is assumed to be ideal. The solving of adiabatic models always needs to cyclic numerical integration and developing of computational programs. The adiabatic model is more actual respect to isothermal model for engines and usually leads to more accurate results. Timumi et al [9] tried to develop the adiabatic method by dividing the engine to five parts. They obtained more accurate results to predict the SE performance in comparison to adiabatic method[9]. Also, many researches have been made to develop and improve the adiabatic analysis considering engine losses[4, 10, 11].

In addition to thermodynamic analysis of SE, numerical methods with high computational cost are introduced recently to evaluate the performance of different part of SE[12-14] .

The motivation of this research is introducing a new method based on Schmidt model to predict actual behavior of SE with low computational cost and accuracy close to adiabatic method. According to authors best knowledge, to analysis the SE performance, no development of Schmidt model based on correcting of effective parameters is done up to now. Also no close-form (explicit) method is presented which can be able to simulate SE with accuracy of adiabatic method. Also, this new approach has been evaluated by GPU-3 SE data.

2. Mathematical model

In this part, structural components and thermo-fluid processes of engine assumed to be ideal and heat input and output power of engine are calculated by corrected equations. To find the correcting functions, the results of adiabatic model are set as the goal and tried to close the results of primary model to them.

The Schmidt model assumes sinusoidal variation for working volume in engine cylinders and represents equations (1-2) to calculate output power and efficiency of engine. Equations of 1-6 explain Schmidt method for SE. The details of Schmidt and adiabatic method are presented in references[5, 15]

$$V_c = V_{cl_c} + \frac{V_{sw_c}}{2}(1+\cos\theta) \quad (1)$$

$$V_e = V_{cl_e} + \frac{V_{sw_e}}{2}(1+cos(\theta+\phi)) \quad (2)$$

$$P = \frac{M.R}{\left[S + \left(\frac{V_{sw_e}\cos\phi}{2\,T_h} + \frac{V_{sw_c}}{2\,T_k}\right)\cos\theta - \left(\frac{V_{sw_e}\sin\phi}{2\,T_h}\right)\sin\theta\right]} \quad (3)$$

Where, S defined as follow

$$S = \frac{V_{sw_c}}{2\,T_k} + \frac{V_{cl_c}}{T_k} + \frac{V_k}{T_k} + \frac{V_r\ln(T_h/T_k)}{(T_h - T_k)} + \frac{V_h}{T_h} + \frac{V_{sw_e}}{2\,T_h} + \frac{V_{cl_e}}{T_h} \quad (4)$$

So, using differentiates of working volume, the expansion and compression works of cycle is calculated.

$$W_e = \pi.P_{mean}.A.V_{sw_e}.\sin\psi / (1+\sqrt{1-A^2})$$

$$W_c = \pi.P_{mean}.A.V_{sw_c}.\sin(\phi-\psi) / (1+\sqrt{1-A^2})$$

$$W_{Cycle} = \pi.P_{mean}.A.V_{sw_e}.\sin\psi.(1-\frac{T_k}{T_h})/(1+\sqrt{1-A^2})$$

In above equations, A, B, ψ and P_{mean} are given by:

$$A = \sqrt{B}/(\frac{T_k}{T_h} + \frac{V_{sw_c}}{V_{sw_e}} + 4.\frac{V_{dead}}{V_{sw_e}}.\frac{T_k}{T_k+T_h}) \quad (8)$$

$$B = \left(\frac{T_k}{T_h}\right)^2 + \left(\frac{V_{sw_c}}{V_{sw_e}}\right)^2 + 2.\frac{T_k}{T_h}.\frac{V_{sw_c}}{V_{sw_e}}.\cos\phi \quad (9)$$

$$\psi = \tan^{-1}\left(\left(\frac{V_{sw_c}}{V_{sw_e}}.\sin\phi\right)\Big/\left(\frac{T_k}{T_h} + \frac{V_{sw_c}}{V_{sw_e}}.\cos\phi\right.\right. \quad (10)$$

$$P_{mean} = P_{Max}.\sqrt{\frac{1-A}{1+A}} \quad (11)$$

Where, θ, ϕ, V_{sw} , V_{cl} , are crank angle, phase angle between compression and expansion spaces, sweep and dead volume respectively. Subscripts of ‘c’ and ‘e’ denote the compression and expansion.

The correcting process is employed to reach results as accurate as adiabatic second order computational methods. To this end, correctness functions are employed.

To define and exact employing of correctness functions, the effective parameters on output power and engine efficiency should be specified. According to equation (7), such parameters are mean working pressure of engine, expanding displacement volume, phase lag angle, temperature ratio and dead volume. As the effects of V_H and P_m for both isothermal and adiabatic models are linear, they cannot be the main source of error for obtained results of isothermal method in comparison with adiabatic method. So correctness functions are considered for three other non-linear parameters.

The suggested correlations for output power and efficiency are as following:

$$W_{Cycle} = \pi . P_m . A . V_H . \sin\theta . \left(1 - \frac{T_C}{T_H}\right) \Big/ \left(1 + \sqrt{1 - A^2}\right) . F(\phi, T, D) \quad (11)$$

$$\eta_{Cycle} = \left(1 - \frac{T_C}{T_H}\right) . G(\phi, T, D) \quad (12)$$

In this equation, F and G are correcting factors which are functions of phase lag angle of engine (ϕ), temperature ratio of cold to hot source (T) and dead to compression volume ratio (D).

To completely cover the variation of different parameters in real engines, in each investigation, five quantitative levels is considered for all three mentioned parameters. So to apply regression to the equation of each correctness function, a sample size of 125 is required. To produce each sample data, at the beginning, the Schmidt equation is solved based on the amount of triple parameters' amounts. Then, with such data the adiabatic model is solved by using numerical code which is developed in MATLAB software. Finally by dividing the adiabatic model result and Schmidt model result, the amount of sample data for each correctness function is obtained. So for obtaining sample data to regression the correctness functions, each of Schmidt and adiabatic model are solved 250 times. Fig.1 shows the mathematical model of presented method schematically. Part of data used in calculating correctness functions listed in table 1.

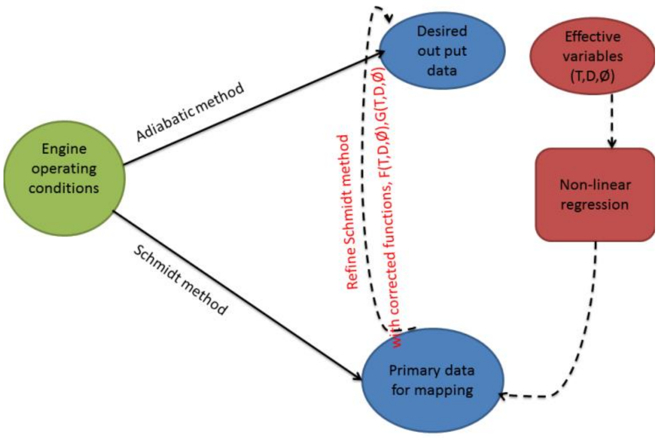

Fig1. Schematic new approach to modeling SE

Table 1.Sample data for calculated correctness functions

NO	T_C [k]	T_H [k]	$T=T_C/T_H$	φ	$D=DV/V_C$	η (Schmidt)	η (Adiab)	W(Schmidt) [W]	W(Adiab) [W]	F	G
1	294	1176	0.25	75	1.0	0.75	0.617	9078.20	9009.21	0.99240	0.82267
2	294	1176	0.25	75	1.3	0.75	0.645	7683.44	8034.37	1.04567	0.86000
3	294	1176	0.25	75	1.6	0.75	0.662	6689.11	7211.83	1.07814	0.88267
4	294	1176	0.25	75	1.9	0.75	0.674	5931.79	6524.97	1.10000	0.89867
5	294	1176	0.25	75	2.2	0.75	0.682	5329.61	5946.15	1.11568	0.90933
121	294	653	0.45	135	1.0	0.55	0.375	3895.34	4577.28	1.17507	0.68182
122	294	653	0.45	135	1.3	0.55	0.407	3251.75	4126.90	1.26913	0.74000
123	294	653	0.45	135	1.6	0.55	0.427	2802.69	3708.07	1.32304	0.77636
124	294	653	0.45	135	1.9	0.55	0.440	2466.16	3347.60	1.35741	0.80000
125	294	653	0.45	135	2.2	0.55	0.449	2202.00	3040.48	1.38078	0.81636

Table2. Sample data for calculated corrected functions (F, G)

NO.	T_C [k]	T_H [k]	$T=T_C/T_H$	φ [degrees]	$D=DV/V_C$	η (Schmidt)	η (Adiab)	W(Schmidt) [W]	W(Adiab) [W]	F	G
1	294	792.0	0.371	111.0	1.67	0.629	0.581	1422.57	1409.91	0.99110	0.92397
2	294	576.7	0.510	102.4	1.93	0.490	0.430	951.29	878.58	0.92357	0.87715
3	294	607.4	0.484	99.2	2.62	0.516	0.470	834.96	793.70	0.95058	0.91086
4	294	568.0	0.518	130.6	1.94	0.482	0.439	716.35	704.23	0.98308	0.91007
5	294	625.0	0.470	122.3	2.55	0.530	0.493	752.79	744.10	0.98846	0.93090
6	294	729.2	0.403	103.1	3.02	0.597	0.562	909.34	896.56	0.98595	0.94170
7	294	535.5	0.549	102.3	2.41	0.451	0.397	732.17	673.77	0.92024	0.88035
8	294	817.9	0.359	94.5	2.21	0.641	0.598	1299.76	1274.71	0.98073	0.93356
9	294	551.2	0.533	92.8	2.49	0.467	0.412	765.81	701.52	0.91605	0.88287
10	294	493.1	0.596	122.5	2.67	0.404	0.362	508.02	479.62	0.94410	0.89645
11	294	820.9	0.358	77.6	2.49	0.642	0.599	1177.9	1141.81	0.96936	0.9333
12	294	711.7	0.413	88.0	2.95	0.587	0.548	929.64	900.28	0.96842	0.93375
13	294	686.6	0.428	94.7	2.13	0.572	0.521	1127.85	1077.31	0.95519	0.91118
14	294	652.0	0.451	107.5	2.39	0.549	0.506	937.31	909.14	0.96995	0.92151
15	294	724.9	0.406	111.3	3.09	0.594	0.563	849.88	844.95	0.99420	0.94713
16	294	538.6	0.546	100.6	2.54	0.454	0.402	717.06	661.92	0.92310	0.88510
17	294	517.5	0.568	88.2	2.64	0.432	0.375	664.89	595.58	0.89576	0.86820
18	294	782.0	0.376	119.8	3.32	0.624	0.598	803.58	811.06	1.00931	0.95824
19	294	609.1	0.483	133.7	1.80	0.517	0.476	787.30	788.70	1.00178	0.92019
20	294	946.6	0.311	96.2	1.92	0.689	0.647	1600.80	1590.45	0.99353	0.93846

3. correctness function

In this research, the regression process is done by using R, Statistical Programming Language. To find the correctness function of output power and engine efficiency, non-linear three parameters Regression is applied. To this end, different models with different structures such as hyperbolic, logarithmic, Exponential and also polynomials with different degrees are evaluated. The result of using these models for regression is compared by considering Least Squares method as criteria. Also a set of 20 random data (Test-Point data) to examine the each reached model.

By different employed investigations, it was clarified that the best description for this research objective functions is presented by polynomial functions. Also the polynomials, which consider the effect of three mentioned parameters interaction, show more desirable results.

More studies showed that increasing the degree of polynomial reduce the Residual Sum of Squares (RSS) favorably which results in almost zero amount for RSS of a suggested model. But by using test-point-data and appearing the over fitting, the performance of this model has been challenged.

Although, the fitted function capture all sample data or close them well, but it can be ignored its smoothness. So in choosing the regression model, both the following sample data and smoothness should be considered.

The set of random test-point-data applied to examine suggested models for correcting power and efficiency of engine with slider and crank mechanism is listed in table 2.

Relative error percentage for each model respect to test-point-data is calculated as follows:

Six suggested model for regression of correcting function of output power are compared and listed in table 3, considering RSS and mean relative error percentage as comparison criteria.

$$\%\,\mathrm{Error} = \left(\frac{1}{n}\sum_{i=1}^{n}\left|\frac{F_{i,\mathrm{Predicted}} - F_{i,\mathrm{Real}}}{F_{\mathrm{Real}}}\right|\right) \times 100 \quad (13)$$

According to the number of effective parameters, the visual representation of under fitting of suggested function is not applicable. But by checking the relative error and least square amounts in table 3, it is evident that increasing the degree of polynomial would deteriorate the results of regression process. The result of relative error in table 3 clearly shows over fitting phenomena models number 5 and 6.

Considering criteria such as matching the results of regression process and sample data, the relative error percentage of regression during the applying of test-point data and the sentences number of polynomial, the model 2 in table 3 is chosen as corrected function fitted for engine output power correlation.

1-2 Quantifying the SE energy losses

To introduce more accurate results, the SE losses are considered in suggested model as follows:

The total pressure loss during the cycle can be obtained from Eq. (14).

$$\Delta P = \Delta P_L + \Delta P_{reg} + \sum \Delta P_m \tag{14}$$

The flow resistance of a component ΔP_L and a pipe $\sum \Delta P_m$ is mention in [15].

The regenerator resistance ΔP_{reg} and average Nusslte number can be calculated from Eq(15) and (16), [14].

$$\Delta P_{reg} = f_{max} \rho L U_{max}^2 / 2D \tag{15}$$

$$Nu_{s-ave} = 8.651 A_0^{0.471} Re_w^{0.361} f_{max}^{0.0401} \tag{16}$$

$$Q_{r,loss} = (1 - \varepsilon)(Q_{r,max} - Q_{r,min}) \tag{17}$$

The heat loss in the regenerator can be calculated as Eq (17) and the heater and cooler actual temperature are calculated as equations (18) [15].

$$Q_h = \frac{60}{n} h_h A_h (T_{wh} - T_h) - Q_{r,loss} \tag{18a}$$

$$Q_k = \frac{60}{n} (h_{k1} A_{k1} + h_{k2} A_{k2})(T_{wh} - T_h) - Q_{r,loss} \tag{18b}$$

The work loss can be calculated from equation (19).

$$W_{loss} = \oint \Delta P_i dV_e \tag{19}$$

3. Results and discussion

In this section, correctness functions are presented. Also, this new method is evaluated by GPU-3 SE data.

Operational characteristics of engines are distinguished by [16]. The other characteristics are geometrical and structural parameters which are constant. The experimental results of GPU-3 engine as specified as [16] are used for validation.

The results of new approach and other analytical models to estimate output power and efficiency of GPU-3 are listed in table4 and compared to experimental data.

Table3. Comparison of suggested fitting modals for corrected function F

Model	Model type	Polynomial degree	Considering of parameters' interaction effect	Number of terms	RSS	Relative error (%)
1	Non-linear	1	✓	8	0.009901	0.6050
2	Non-linear	2	✓	27	0.000438	0.1173
3	Non-linear	3	✓	64	0.000254	0.0506
4	Non-linear	4	✓	125	0.000038	0.0868
5	Non-linear	5	✓	216	0.000009	42.62
6	Non-linear	6	✓	343	0.000001	303.59

Table4. Results of different models and experimental data for GPU-3

model	Output power (W)	Engine efficiency (%)
Adiabatic model	8286	62.0
Extended adiabatic model by Urieli and Berchowitz 17]	8300	62.5
Timumi model (without considering engine loss)[18]	7109	54.9
Semi-steady model of Urieli et al. [19]	7400	53.1
Urieli et al. model (considering pressure loss) [19]	6700	52.5
Timumi model (considering engine loss)[18]	4273	38.5
Present model*	4150	37.7
Experimental data of GPU-3 engine[16]	3958	35.0

Conclusion

In this research, a new method to investigate the output power and efficiency of real engines is presented. In this explicit approach, without using numerical methods and iterative solver programs, comparative results with accuracy and validity of adiabatic numerical method are obtained. By applying correctness functions and using engine energy losses correlations, the evaluating of output power and thermal efficiency of real engines has been applicable as a set of correlations with simple close-form solution. The compared results showed good agreement with error less than 3% for thermal efficiency prediction.

References

[1]. S. Tyagi, S. Kaushik, and M. Singhal, "Parametric study of irreversible Stirling and Ericsson cryogenic refrigeration cycles," Energy conversion and management, vol. 43, pp. 2297-2309, 2002.

[2].] F. Formosa and G. Despesse, "Analytical model for Stirling cycle machine design," Energy Conversion and Management, vol. 51, pp. 1855-1863, 2010.

[3].] H. Solmaz and H. Karabulut, "Performance comparison of a novel configuration of beta-type Stirling engines with rhombic drive engine," Energy Conversion and Management, vol. 78, pp. 627-633, 2014.

[4]. C. J. Paul and A. Engeda, "Modeling a complete Stirling engine," Energy, vol. 80, pp. 85-97, 2015.

[5]. G. Schmidt, "Classical analysis of operation of Stirling engine," A report published in German engineering union (Original German), vol. 15, pp. 1-12, 1871.

[6]. I. Urieli and D. Berchowitz, "Stirling cycle machine analysis," Bristol Hilger Ltd, 1984.

[7]. S. KILDEGARD ANDERSEN, H. Carlsen, and P. GROVE THOMSEN, "Preliminary

[8]. results from simulations of temperature oscillations in stirling engine regenerator matrices," Energy, vol. 31, pp. 1371-1383, 2006.

[9]. I. Batmaz and S. Üstün, "Design and manufacturing of a V-type Stirling engine with double heaters," Applied Energy, vol. 85, pp. 1041-1049, 2008.

[10]. Y. Timoumi, I. Tlili, and S. B. Nasrallah, "Performance optimization of Stirling engines," Renewable Energy, vol. 33, pp. 2134-2144, 2008.

[11]. J. Wrona and M. Prymon, "Mathematical modeling of the stirling engine," 2016.

[12]. M. Mahmoodi and M. Ziabasharhagh, "Numerical solution of beta-type Stirling engine by optimizing heat regenerator for increasing output power and efficiency," J Basic Appl Sci Res, vol. 2, pp. 1395-1406, 2012.

[13]. W.-L. Chen, K.-L. Wong, and Y.-F. Chang, "A computational fluid dynamics study on the heat transfer characteristics of the working cycle of a low-temperature-differential γ-type Stirling engine," International Journal of Heat and Mass Transfer, vol. 75, pp. 145-155, 2014.

[14]. J. L. Salazar and W.-L. Chen, "A computational fluid dynamics study on the heat transfer characteristics of the working cycle of a β-type Stirling engine," Energy Conversion and Management, vol. 88, pp. 177-188, 2014.

[15]. H. Saberinejad and A. Keshavarz, "Reciprocating turbulent flow heat transfer enhancement within a porous medium embedded in a circular tube," Applied Thermal Engineering, vol. 102, pp. 1355-1365, 2016.

[16]. S. Alfarawi, R. AL-Dadah, S. Mahmoud, Influence of phase angle and dead volume on gamma-type Stirling engine power using CFD simulation. Energy Conversion and Management 124 (2016) 130-140.

[17]. W. R. Martini, Stirling engine design manual, Washington Univ., Richland (USA). Joint Center for Graduate Study, pp. 1978 .

[18]. I. Urieli, C. J. Rallis, D. M. Berchowitz, Computer simulation of Stirling cycle machines, in Proceeding of, 1512-1521 .

[19]. Y. Timoumi, I. Tlili, S. B. Nasrallah, Design and performance optimization of GPU-3 Stirling engines, Energy, Vol. 33, No. 7, pp. 1100-1114, 2008 .

[20]. I. Urieli, C. J. Rallis, D. M. Berchowitz, Computer simulation of Stirling cycle machines, in Proceeding of, 1512-1521 .