

Where f_j is the activation function of neuron j and I_j is the sum of weighted input nodes of neuron j that is calculated in (2).

$$I_j = \sum_{i=1}^{N_i} w_{j,i} y_{j,i} + w_{j,0} \tag{2}$$

Where $y_{j,i}$ is the i th input node of neuron j weighted by $w_{j,i}$, and $w_{j,0}$ is the bias weight of neuron j . The goal of the training algorithms is the adjustment of the input node and bias weights, so that the considered cost function is located in its minima.

3. Levenberg-Marquardt Algorithm

3.1. Optimization Problem

The Levenberg–Marquardt (LM) algorithm [22, 23], which was independently developed by Kenneth Levenberg and Donald Marquardt, provides a numerical solution to the problem of minimizing a nonlinear function. It is fast and has stable convergence. In the artificial neural-networks field, this algorithm is suitable for training small and medium-sized problems.

The LM algorithm blends the steepest descent method and the Gauss–Newton algorithm. Fortunately, it inherits the speed advantage of the Gauss–Newton algorithm and the stability of the steepest descent method. It’s more robust than the Gauss–Newton algorithm, because in many cases it can converge well even if the error surface is much more complex than the quadratic situation. Although the LM algorithm tends to be a bit slower than Gauss–Newton algorithm (in convergent situation), it converges much faster than the steepest descent method [24].

The basic idea of the LM algorithm is that it performs a combined training process: around the area with complex curvature, this algorithm switches to the steepest descent algorithm, until the local curvature is proper to make a quadratic approximation; then it approximately becomes the Gauss–Newton algorithm, which can speed up the convergence significantly [25].

The problem of LM provides a solution is called nonlinear least square minimization. This implies that the function to be minimized of the following special form:

$$f(x) = \frac{1}{2} \sum_{j=1}^m r_j^2(x) \tag{3}$$

Where $x = (x_1, x_2, \dots, x_n)$ is a vector and each r_j is a function from \mathfrak{R}^n to \mathfrak{R} . The r_j s are referred to as residuals and it is assumed that $m \geq n$. To make matters easier, f is represented as residual vector $r: \mathfrak{R}^n \rightarrow \mathfrak{R}^m$ defined by (4)

$$r(x) = (r_1(x), r_2(x), \dots, r_m(x)) \tag{4}$$

Now, f can be rewritten as $f(x) = \frac{1}{2} \|r(x)\|^2$. The derivatives of f can be written using the Jacobian matrix J of r w.r.t x defined as (5)

$$J(x) = \frac{\partial r_j}{\partial x_i}, 1 \leq j \leq m, 1 \leq i \leq n \tag{5}$$

If every r_i function is considers as linear, the Jacobian is constant and r can be presented as a hyper-plane through space. So that f is given by the quadratic form as shown in (6), also $\nabla f(x)$ and $\nabla^2 f(x)$ is calculated by (7) and (8), respectively.

$$f(x) = \frac{1}{2} \|Jx + r(0)\|^2 \tag{6}$$

$$\nabla f(x) = J^T (Jx + r) \tag{7}$$

$$\nabla^2 f(x) = J^T J \tag{8}$$

Solving for the minimization by setting $\nabla f(x) = 0$, we obtain x_{\min} as (9)

$$x_{\min} = -(J^T J)^{-1} J^T r \tag{9}$$

Which is the solution to the set of normal equations.

3.2. LM as a blend of Gradient descent and Gauss-Newton Iteration

Gradient descent is the simplest, most intuitive technique to find minima in a function. Parameter updating is performed by adding the negative of the scaled gradient at each step, as (10).

$$x_{i+1} = x_i - \lambda \nabla f \tag{10}$$

Simple gradient descent suffers from various convergence problems. Logically, we should like to take large steps down the gradient descent at locations where the gradient is small and conversely, take small steps when the gradient is large, so as not to rattle out of the minima. With

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the above update rule, we do just the opposite of this. Another issue is that the curvature of the error surface may not be the same in all directions. For example, if there is a long and narrow valley in the error surface, the component of the gradient in the direction that points along the base of the valley is very small while the component along the valley walls is quite large. This results in motion more in the direction of the walls even though we have to move a long distance along the base and a small distance along the walls [26].

This situation can be improved upon by using curvature as well as gradient information, namely second derivatives. One way to do this is to use Newton's method to solve the equation $\nabla f(x) = 0$. Expanding the gradient of f using a Taylor series around the current state x_0 , $\nabla f(x)$ is calculated as (11).

$$\nabla f(x) = \nabla f(x_0) + (x - x_0)^T \nabla^2 f(x_0) + H.O.T \quad (11)$$

If we neglect the higher order terms (H.O.T) and solve for the minimum by setting the left hand side of (9) to zero, we get the update rule for Newton's method, as (12)

$$x_{i+1} = x_i - (\nabla^2 f(x_i))^{-1} \nabla f(x_i) \quad (12)$$

Where, x_0 has been replaced by x_i and x by x_{i+1} .

Since Newton's method implicitly uses a quadratic assumption on f , the Hessian need not be evaluated exactly. The main advantage of this technique is rapid convergence. However, the rate of convergence is sensitive to the starting location (or more precisely the linearity around the starting location). It can be seen that simple gradient descent and Gauss-Newton iteration are complementary in the advantages they provide. Levenberg proposed an algorithm based on this observation [22], whose update rule is a blend of the above mentioned algorithms and is given as (13)

$$x_{i+1} = x_i - (H + \lambda I)^{-1} \nabla f(x_i) \quad (13)$$

Where H is Hessian matrix evaluated at x_i . This update rule is used as follows. If the error goes

down following an update, it implies that our quadratic assumption on $f(x)$ is working and we reduce λ (usually by a factor of 10) to reduce the influence of gradient descent. On the other hand, if the error goes up, we would like to follow the gradient more and so λ is increased by the same factor.

It is to be noted that while the LM method is in no way optimal but is just a heuristic, it works extremely well in practice. The only flaw is its need for matrix inversion as part of the update. Even though the inverse is usually implemented using clever pseudo-inverse methods such as singular value decomposition [27], the cost of the update becomes prohibitive after the model size increases to a few thousand parameters. For moderately sized models (of a few hundred parameters) however, this method is faster than gradient descent. The block-diagram for training using LM algorithm is shown in Fig. 2.

4. Implementation and Experimental Results

4.1. Experimental Setup

Experimental setup used in this paper is shown in Fig. 3. This work is done in Advanced Vehicle Control Systems laboratory (AVCS lab), at K.N. Toosi University of Technology.

The MEMS-based accelerometer is placed in the vehicle and an incremental encoder, whose resolution is 200 pulse per revolution, is connected to the rear wheel of the vehicle. This encoder can determine the vehicle's position by the accuracy of approximately 1cm. The acceleration measured by encoder output is considered as the reference signal.

The proposed inertial accelerometer calibration method can be used in the structure of advanced driver assistance systems such as brake assist and collision avoidance systems to estimate the vehicle acceleration.

Table1. Performance comparison of the proposed method with Newton and GD methods.

Training Method	RMSE	Number of Iterations
Newton	0.0326	28
Gradient Descent (GD)	0.0893	1000 (Maximum)
Levenberg-Marquardt (LM)	0.0318	12

5. Conclusion

Measurement of vehicle acceleration is one of the most important task to design of driver assistance systems and autonomous vehicles. Low-cost inertial MEMS-based sensors are widely used in the structure of vehicle navigation, but the error sources in the output of these sensors could have disruptive effects on the results. In this paper, a calibration algorithm based on neural network trained by Levenberg-Marquardt was suggested to remove these effects. Results showed that presented method has an acceptable performance in real driving situations. The presented method can estimate the vehicle acceleration and is beneficial to use in the structure of advanced driver assistance systems.

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