Obtaining relations between the Magic Formula coefficients and tire physical properties

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Abstract

This paper introduces a technique that relates the coefficients of the Magic Formula tire model to the physical properties of the tire. For this purpose, the tire model is developed by ABAQUS commercial software. The output of this model for the lateral tire force is validated by available tire information and then used to identify the tire force properties. The Magic Formula coefficients are obtained from the validated model by using nonlinear least square curve fitting and Genetic Algorithm techniques. The loading and physical properties of the tire such as the internal pressure, vertical load and tire rim diameter are changed and tire lateral forces for each case are obtained. These values are then used to fit to the magic formula tire model and the coefficients for each case are derived. Results show the existing relationships between the Magic Formula coefficients and the loading and the physical properties of the tire. In order to investigate the effectiveness of the method, different parameter values are selected and the lateral force for each case are obtained by using the estimated coefficients as well as with the simulation and the results of the two methods are shown to be very close. This proves the effectiveness and the accuracy of the proposed method.

***Keywords**: Magic formula, MF, Pacejka, tire model, Genetic algorithm, Least squares*

1. Introduction

The pneumatic tires carry the vehicle weight, reduce the vertical motions on irregular surfaces and supply sufficient forces to accelerate or decelerate the vehicle. Tire cornering force holds the vehicle on a corner and stabilizes the vehicle motion. Estimating the tire-road contact forces is, therefore, the first step in the vehicle handling and safety analysis.

In the vehicle engineering, describing the tire forces is a complex problem. The reason is the complexity of the tire construction, geometry and contact properties that make it hard to model these forces by using the fundamental and simple lows of physics. The tire forces have non-linear and complex relation with quantities associated with the tire such as geometric measures and physical properties, tire pressure, temperature, friction, vertical forces, longitudinal and lateral slips and more.

Most of the literature concerning the tire behavior deals with the quasi-static part of the tire response in which steady forces and moments result from inputs such as slip or camber angle as the tire is rolling. A large number of quasi-static tire models have been proposed which include theoretical (or physically based), empirical and semi-empirical models. Among the several useful quasi-static tire models, the semi-empirical Magic Formula (MF) tire model [1, 2] can be regarded as the best and most comprehensive model for use in vehicle dynamics studies. In fact this model has now been widely accepted and used throughout industry. This is due to its high level of correlation with tire test data and in addition, its ability to be implemented relatively simply into a vehicle simulation program. Moreover, its parameters have physical meanings that are useful for the characterization of tire shear force generation properties. However, difficulties remain in using the model due to the volume of test data needed for a specific tire and also the process of finding the coefficients of the formulae by curve fitting methods. All these techniques have been able to find relations for the formula factors by using measured results for a specific tire. In addition, in most of these methods, complex relations and calculations are employed to obtain the factors for the tire model.

Finite Element Models (FEM) are also useful methods in modeling the behavior of tires. The tire contact problem is one of important issues studied by several researchers such as Noor and Tanner [3]. In their work in a NASA research program for the space shuttle tires, the status and developments of the computational models for tires are summarized. A similar review has also been made again by Danielson [4]. ABAQUS is a commercial software that several researchers have found it useful for tire contact force modeling. Examples are the works of Oden on friction and rolling contact [5], Laursen and Simo in the field of contact problems with friction [6], Padovan on rolling visco-elastic materials [7] and Wriggers on constitutive interface laws with friction [8].

Basically all early tire models were axi-symmetric, with no tread pattern or only circumferential groves due to the limits of computational resources. With the ever increasing computational power it is now possible to mesh a part or even the whole tread with a detailed pattern (e.g. Cho [9]) and perform all kinds of static analysis such as footprint shapes as functions of the axle load state and pressure. In an article written by Kabe and Koishi [10] a comparison between ABAQUS standard, explicit and experiments for steady state cornering tires was made. The results of both implicit and explicit methods were closer to each other than to the experiments. The implicit method, however, was significantly faster. The prediction of tire cornering forces is discussed by Tonuk and Unlusoy [11], where a comparison with experiments is also presented. The application of a type of non-linear 3D finite element tire model for simulating tire spindle force and moment response during side slip is described by Darnell [12]. The simulation model is composed of shell elements, which model the tread deformation, coupled to special purpose finite elements that model the deformation of the sidewall and contact between the tread and the ground. Despite the simple model the results corresponds quite well with experiments.

A simulation with ABAQUS is studied by Olatunbosun and Bolarinwa and the effect of tire design parameters on lateral forces and moments [13] is presented. Parametric studies are performed on a simplified tire with no tread, negligible rim compliance and viscoelastic properties, shear forces modeled with Coulomb friction. Explicit simulations to predict tire cornering forces are presented by Koishi [14]. Besides a comparison with experiments, parametric studies on the effect of inflation pressure, belt angle and rubber modulus are performed.

The empirical methods based on testing to obtain the tire forces, have the disadvantage of repeating the costly and time consuming tests, once the tire physical properties need be changed. On the other hand, in obtaining the factors of the Magic Formula the idea of establishing relations based on the physical properties of a tire has not been studied. This paper aims at finding relations for the main coefficients of the Magic Formula tire model with the physical properties so that one can express the tire forces without having to perform several expensive and time consuming tests. For this purpose a complex FEM tire model will be used and validated in order to produce a large number of test results when the physical properties of the tire are changed. In this paper an optimum method to have precise results and less analyzing time is applied.

2. Methodology

Among several methods to estimate the tire forces, the Magic formula model is widely employed in the analysis and modeling of the vehicle dynamics. This formula was introduced in 1987 by Pacejka and his colleagues [1], but it was improved over the years and various versions are available. This formula is a semi-empirical model based on several expensive experimental tests and software methods to fitting the formula to the test data. The formula contains a number of coefficients that are functions of the effective parameters. The magic formula has been able to express the tire behavior in such a way that the researchers used it extensively. It can be argued, therefore, that possible relations between the magic formula coefficients and the physical properties of a tire such as dimension, material, pressure and other factors are expected to exist.

In this paper, it is intended to analyze the possible relations between the Pacejkaبs MF tire coefficients and its physical properties, by using a particular methodology. The proposed method is divided into two parts. At first, a model of the tire is constructed in the Abacus software in order to obtain the tire-road contact forces. In the second part, by obtaining the tire forces when the physical properties are varied, the data is fitted to the MF equation and the dependence of the coefficients on the physical parameters is analyzed.

3. The Magic Formula model

This model represents uncoupled longitudinal and lateral forces, and self-aligning moment of the tire by a unique formula. The model also provides a method of dealing with the problem of combined cornering and braking (or acceleration) by making use of the uncoupled force/moment formula and some additional mathematical expressions. Although for the general non-linear vehicle models both the tractive and cornering forces generated by tires are considered, for the sake of simplicity, only the uncoupled lateral force model is used.

The equations described here for the Magic Formula representation of a tire are taken from the 1987 version [1]. This version was, however, later updated, e.g. in 1989 [2] but since the later version did not contain accompanying data, it was decided to use the earlier version for the current study. The Magic Formula (MF) representation of the tire force/moment equation is of the basic form [1],

$$Y(X) = D \quad \sin(C \quad \tan^{-1}(B\Phi)) + S_v \tag{1}$$

Where

$$B\Phi = G(1-E) + E \quad \tan^{-1}(G) \tag{2}$$

$$G = B(X + S_h) \tag{3}$$

Y stands for either side force F_y , self-aligning moment M_z, or braking/acceleration force F_x. X in Eq. (1) represents either slip angle α or longitudinal slip s_x . The coefficients B, C, D, E, and the horizontal and vertical shift functions S_h and S_v depend on the vertical tire load F_z and camber angle γ.

The non-linear equations for the coefficients of the cornering force F_y are,

$$C_y = 1.3 \tag{4}$$

$$D_y = a_1 F_z^2 + a_2 F_z \tag{5}$$

$$bcd_y = a_3 \sin[a_4 \tan^{-1}(a_5 F_z)](1 - a_{12}|\gamma|) \tag{6}$$

$$B_y = \frac{bcd_y}{C_y \times D_y} \tag{7}$$

$$E_y = a_6 F_z^2 + a_7 F_z + a_8 \tag{8}$$

$$S_h = a_9 \gamma \tag{9}$$

$$S_v = (a_{10} F_z^2 + a_{11} F_z)\gamma \tag{10}$$

where a1 ت a11 are constants and for a given tyre must be determined through a curve fitting procedure. The typical variations of the lateral force Fy versus the lateral slip angle is illustrated in Figure 1.

2.2 Estimating the MF factors

This idea is based on the fact that when the MF formula can express the tire behaviors with a good accuracy, so the physical properties of the tire should be influential in the MF factors. With this in mind, if the relation between the MF constants and tire physical properties were available, then the tire forces could be obtained easily by using the tire physical characteristics. In other words, one can produce the MF for a given tire once the physical properties are obtained.

To this end, relations must be obtained between the MF factors and the physical properties of tires such as geometrical (e.g. radius, width, aspect ratio) and material (e.g. belts, rubber) properties. The result is then of the form:

$$F_i = F_i(p_1, p_2, \cdots) \tag{11}$$

where Fi stands for the MF factors B, C, D and E, whereas p1, p2, etc are the physical parameters.

This can be performed by first developing a detailed physical model validated against the test data or the MF tire model curves, and then obtaining a large number of output results at each property changes. This paper focuses on obtaining the variation of each factor when the important physical properties are changed.

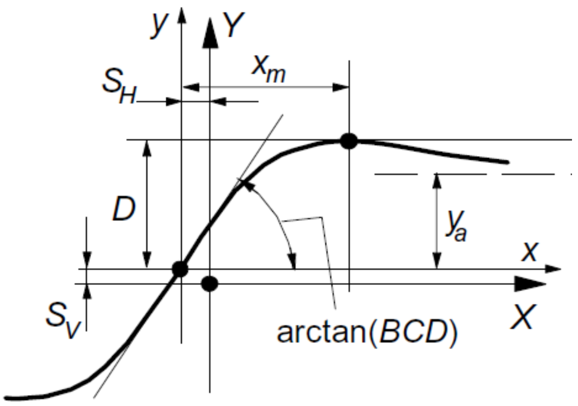

Fig1. The tire lateral force and MF parameters [15]

3. Tire Modeling

One of the available methods for tire-road contact surface modeling and obtaining the tire forces is modeling in the well-established FEM software such as ABAQUS. The tire model is developed by applying the different boundary conditions. When the model is valid, then different tire performances can be obtained by using this tire model.

In this paper, the Radial tire P205/60R14 is modeled under various loading conditions. For the modeling by using ABAQUS software, at first a 2-dimensional axisymmetric model of a tire is designed and then for having a 3-dimensional model, the 2-dimensional model is rotated around the tire rotation axis. In this modeling, tire components are considered as similar as possible to the actual tire parts. The simulating tire model in this paper includes a tire body, two metallic belts and two nylon belts. The robber is described by a non-compressible hyper-elastic material with Moony Rivlin Model [16]. In the process of loading, at the beginning the tire is installed on the ring and then to simulate internal air pressure of the tire, 248KPa internal pressure is applied inside the tire. For the next step, for modeling car weight 4000N vertical force is applied to the air axle. The road modeling is performed by using a rigid plate. In the basic model, the friction of contact area is considered zero, but in analyzing of stretching and breaking, the Coulomb friction model is used. In the final loading, where the rotation is stable, the ground speed is considered 10km/h [17]. The tire model in the software environment is illustrated in Figure 2.

3.1 Model validation

The results of the model must be validated against those of the MF model. As shown in Figure 3 for the lateral force, for the slip angles above 5 degrees the full compliance doesn't happen because of assuming a Coulomb friction model and neglecting the details of the tire tread friction in the contact area. In order that the results of the modeling exactly match those of the MF model, by changing the factors of tire contact friction model, the model output is exactly adjusted to the MF graph as shown in Figure 4. The validity of this friction model is assured for different vertical forces as depicted in Figure 5. Therefore the resulting model can be considered valid as it generates results very close to those of the MF model at different tire normal forces at the full range of slip angles.

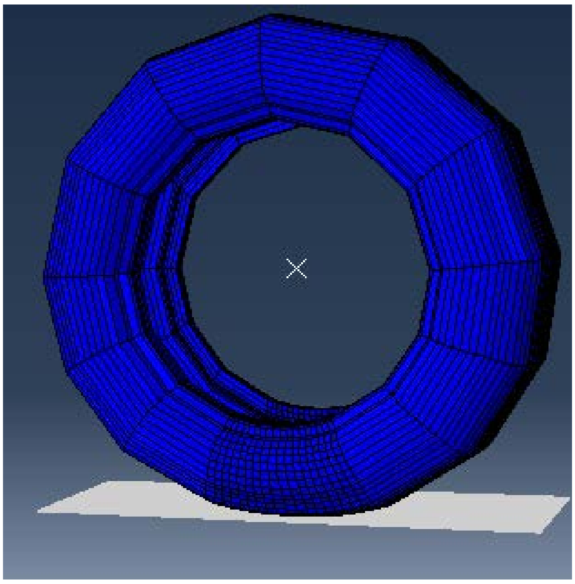

Fig2. Tire model in ABAQUS software

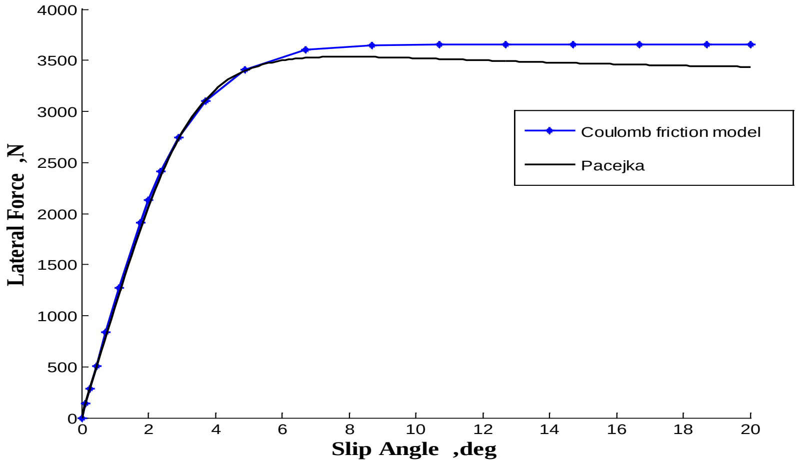

Fig3. Variation of the lateral force vs slip angle for the Coulomb friction model

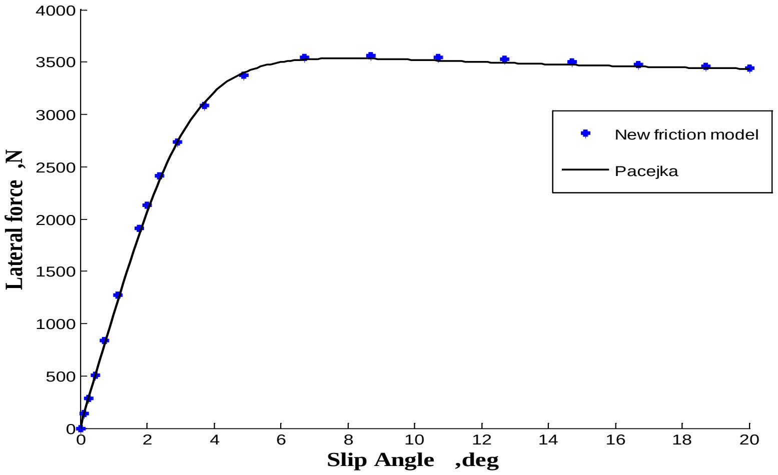

Fig4. Validation of the lateral force- slip angle by adjusting the friction model

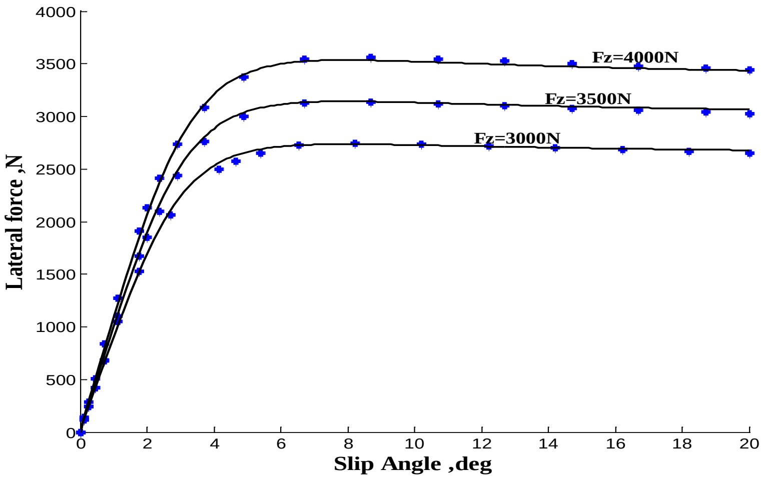

Fig5. The variation of lateral force- slip angle at different vertical forces

4. Estimating MF constants

In the previous section, the lateral force generation of the tire P205/60R14 was modeled in the ABAQUS software and validated against the MF tire model. The base values of the tire are provided in Table 1.

Once the variation of the tire force with the slip angle is simulated by using the model, the resulted data can be used to determine the MF model coefficients. For this purpose, one of the curve fitting techniques can be applied. Among several options, the genetic algorithm (GA) and non-linear least-squares (LS) methods are used here. The results of fitting data to the MF formula by making use of the GA and LS methods are compared with the original MF model values in Table 2. Although there are some differences between the GA method and LS approaches especially for E factor, the resulting lateral force diagrams obtained by the MF formula for the both methods are almost the same as shown in Figure 6.

The Genetic algorithm technique is more difficult to work with compared to the LS method. It was applied here for the comparison purposes. For the rest of the work, however, the faster method namely the non-linear least-squares technique will be used for estimating the MF formula for tires with other features.

Table 1. The base values of tire parameters

	Parameter	Value	Unit
1	Tire normal load	4000	N
2	Internal pressure	248	kPa
3	Young's Modulus for Rubber	6800	N/mm2
4	Young's Modulus for Nylon belts	1950	N/mm2
5	Young's Modulus for Steel belts	198700	N/mm2
6	Angle of belts	20	deg

Table 2. Comparing the factors of the MF function.

Parameter Method	B	E	C	D	Sh	Sv
MF model	0.250	-1.60	1.22	3536	0	0
Genetic Algorithm	0.249	-1.60	1.22	3536	0.027	0.01
Least squares	0.257	-0.714	1.26	3566	0	0

5. Effects of changes in the physical parameters

Among the effective physical parameters of a tire on its force generation mechanism, the following parameters are more influential:

The tire size (e.g. the radius of the ring, the aspect ratio, the tire profile)

Tread properties (material and tread design)

Properties of reinforcement cords (material, number, size and cross section area, the distance between straps and alignment angles).

The air pressure inside the tire

Tire vertical load

Among the above cases, the tire internal air pressure, vertical force and the ring radius have larger impacts on the tire force and in this paper the changes of the MF constants due to the changes of these parameters are analyzed.

6. The results

In this section, the changes of the MF constants due to the changes of selected parameters namely the tire pressure, vertical load and the rim radius are studied. For this purpose, at each stage, other features of a tire are kept constants and just one of the properties is altered. Then the lateral force and the MF coefficients are obtained at each case.

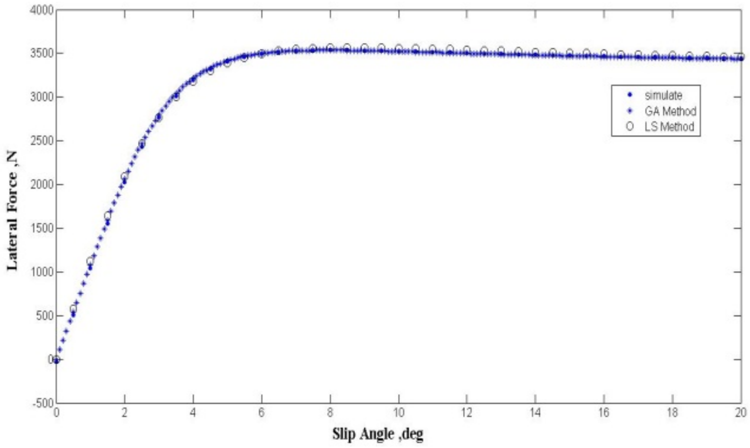

Fig6. The lateral force variation of MF formula, compared for the three methods

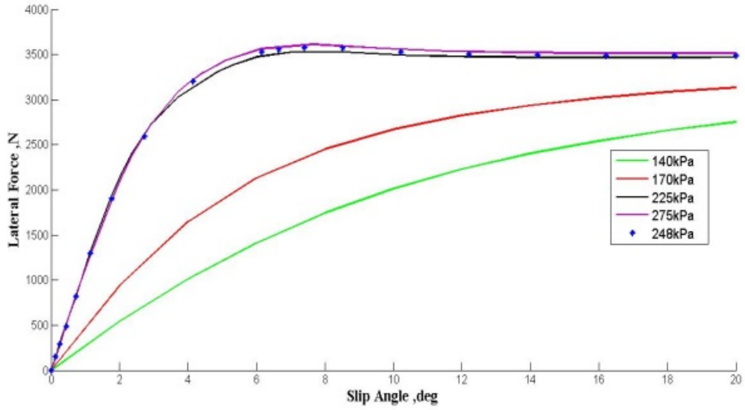

Fig7. The changes of the tire lateral force with changes of internal pressure

6-1- Effects of tire pressure

By changing the internal pressure of the tire, the lateral force variation is obtained and then by applying the non-linear least-squares technique the values of the MF coefficients are estimated. In this way the influence of the tire pressure on the variations of the MF coefficients is observed. Five values of tire pressure namely 140, 175, 225, 248 and 275kPa (corresponding to 20, 25, 30, 35 and 40 psi respectively) are simulated and the results are depicted in Figure 7. It is observed that the tire force undergoes large changes when the pressure is dropped to 25 psi or more.

After producing the lateral force diagrams, these graphs are fitted to the MF model and the coefficients B, C, D and E are determined for each tire pressure. The factors Sh and Sv in all cases were considered as zeros. The LS technique is used for this curve fitting and the results are plotted against the tire pressure values as shown in Figure 8. The graphs show the changes of the MF constants due to the changes of the tire pressure. In order to observe the correctness of the results, the pressure value of 200kPa is considered that is not one of the simulated pressures and the MF constants is obtained by using the graphs of Figure 8 and is shown in Table 3. On the other hand, for the tire with pressure 200kPa a simulation is performed and its lateral force diagram is plotted in Figure 9. In the same diagram the result generated by the MF with the coefficients of Table 3 is also presented. The two results are close to a good extent and the fitness method proves acceptable.

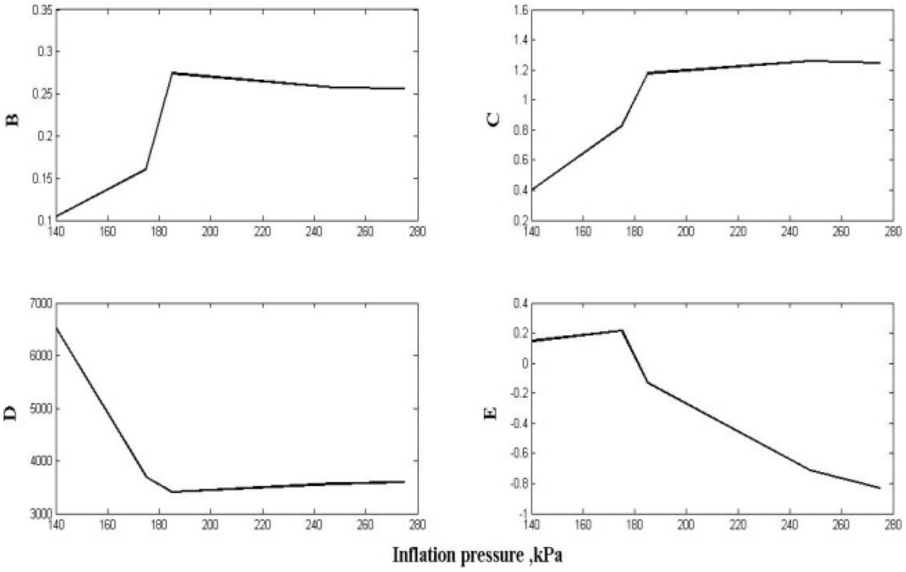

Fig8. The variation of the MF coefficients vs the changes of tire pressure

Table 3. MF constants generated from Figure 8 for tire pressure of 200kPa

Parameter Pressure	B	E	C	D	Sh	Sv
200 kPa	0.285	-0.28	1.2	3480	0	0

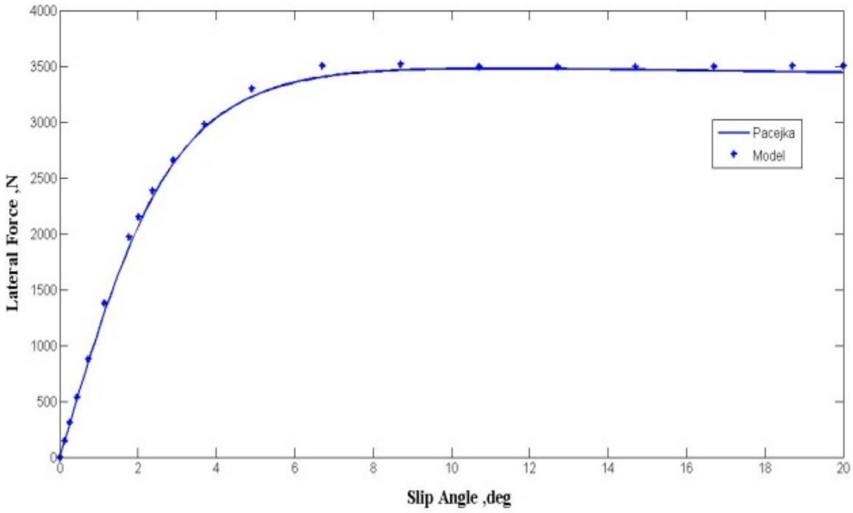

Fig9. Comparing the results of MF model with coefficients of Table 3 and simulation result at 200kPa pressure

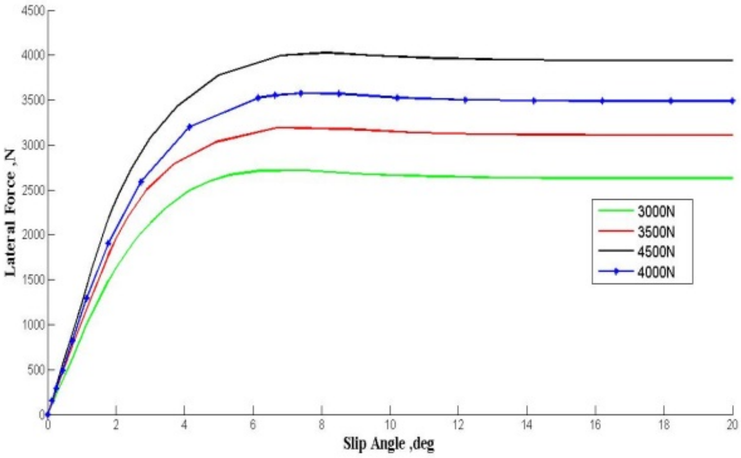

Fig10. The changes of the tire lateral force due to changes of the vertical load.

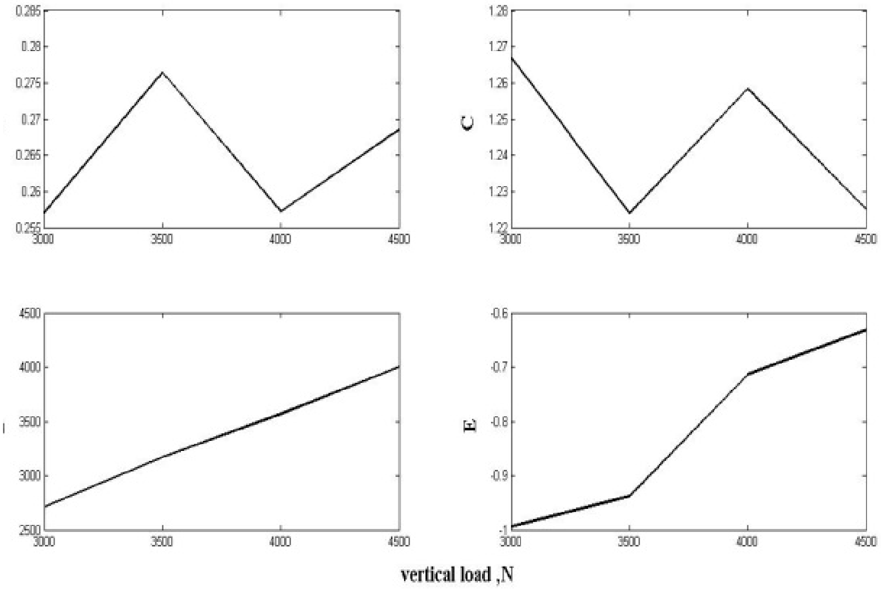

Fig11. The variation of the MF coefficients vs the changes of the vertical force

Table 4. MF constants generated from Figure 11 for the vertical load of 3250 N

Parameter Vertical load	B	E	C	D	Sh	Sv
3250 N	0.267	-0.97	1.246	2950	0	0

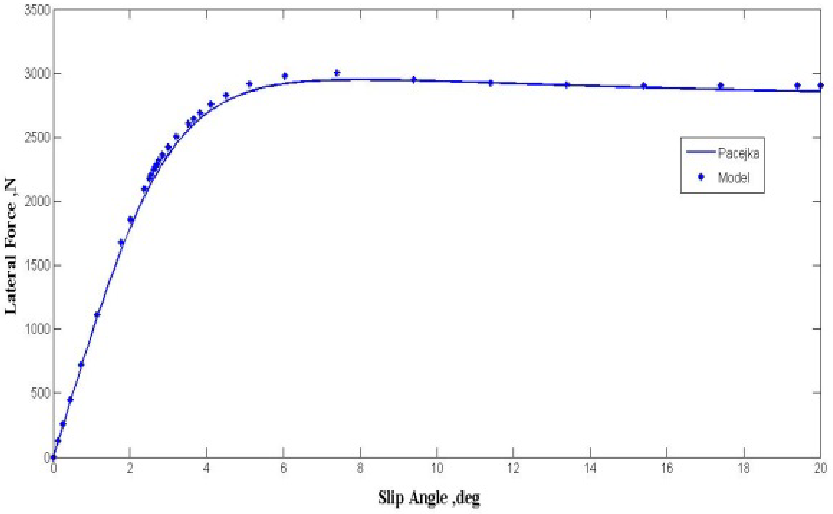

Fig12. Comparing the results of MF model with coefficients of Table 4 and simulation result at 3250 N vertical force

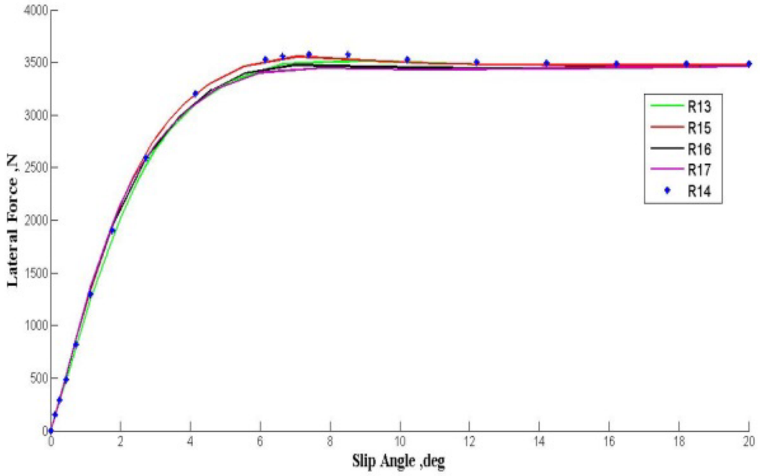

Fig13. The changes of the tire lateral force due to the changes of the tire ring diameter

6-2- Effect of vertical load

The graph of the changes of the tire lateral force with the vertical loads of 3000, 3500, 4000 and 4500 N by using the simulation is presented in Figure 10. The variation of the MF constants owing to the changes of the vertical force is presented in Figure 11. The MF constants are estimated by LS technique. Similar to the previous case, for the vertical load of 3250 N, the MF constants shown in Table 4 are determined by using the information of Figure 11. The results for the same vertical load obtained by the ABAQUS simulation is shown in Figure 12 together with that determined by the MF model and the coefficients of Table 4. The two outputs are close to a good extent.

6-3- Effect of rim diameter

Figure 13 shows the variation of the lateral force for tires with the different diameters of 13, 14, 15, 16 and 17 inches generated by the simulation. It can be seen that the ring diameter does not affect the lateral force considerably. Figure 14 shows the changes of the MF constants based on the changes in the ring diameter by using the LS technique. For the ring radius 16", the MF constants are calculated from these diagrams and are shown in Table 5.

The variation of the tire force obtained by the MF equation, with the constants of Table 5 and also the result of simulation by ABAQUS software for the ring diameter of 16پ are shown in Figure 15. Both results are close and thus it is concluded that the estimation of the MF coefficients is performed with an acceptable precision.

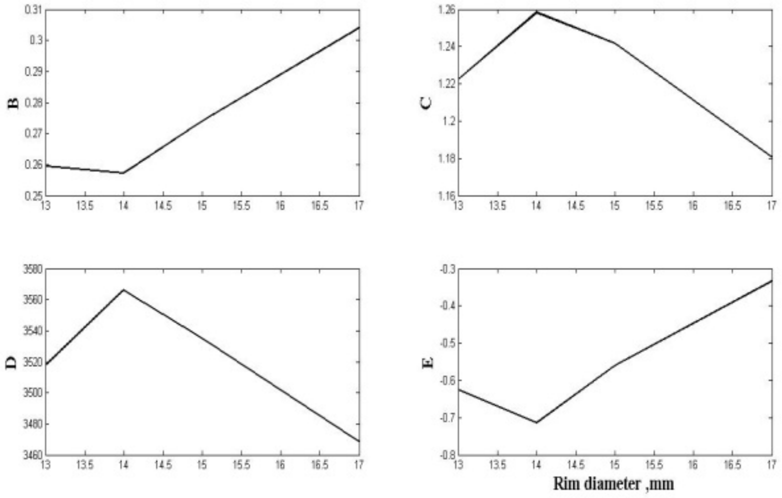

Fig14. The changes of the MF constants vs the changes of the ring diameter

Table 5. MF constants generated from Figure 14 for the ring diameter of 16"

Parameter Ring diameter	B	E	C	D	Sh	Sv
16پ	0.289	-0.45	1.21	3520	0	0

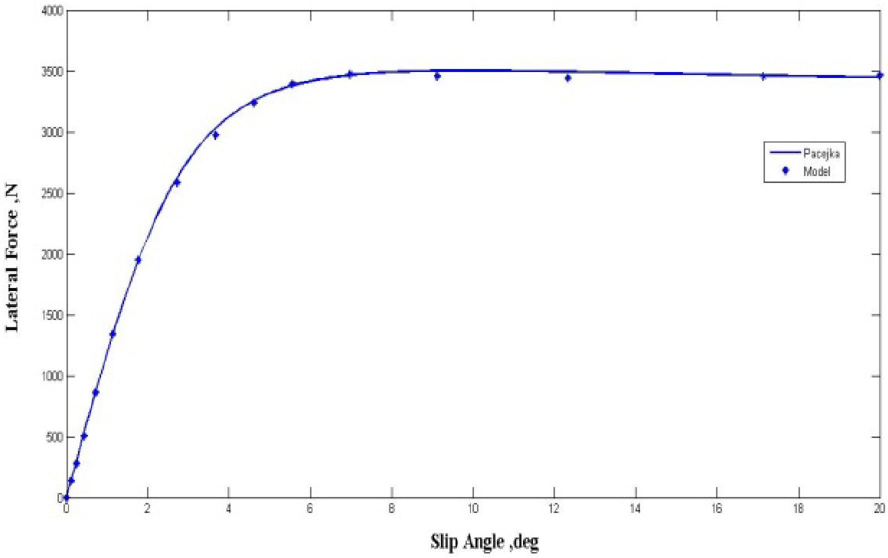

Fig15. Comparing the results of MF model with coefficients of Table 5 and simulation result for the ring diameter 16"

The variation of the tire force obtained by the MF equation, with the constants of Table 5 and also the result of simulation by ABAQUS software for the ring diameter of 16ﭙare shown in Figure 15. Both results are close and thus it is concluded that the estimation of the MF coefficients is performed with an acceptable precision.

7. Conclusions

In this paper a new technique is presented that makes possible to relate the MF tire model coefficients to the tire physical features such as vertical force, internal pressure, the diameter of the tire ring and other parameters. In order to produce reliable tire lateral force results, a model is developed by using ABAQUS commercial software. The results of this model are validated with those of Magic Formula tire model.

The validated model was used to study the variation of the tire lateral force when the loading of tire or its physical parameters are change. For each case, the results are first fitted to the MF tire model and the coefficients are estimated by using non-linear Least-square method. The Genetic algorithm technique was also used for comparison purposes. The results of changing the tire properties lead to MF model coefficients represented as functions of the tire parameters.

It was shown that this technique was able to estimate the MF tire model coefficients in terms of the tire loading and physical properties. The precision of this technique was shown to be good compared to the validated simulation results. Further developments of this technique can provide a tire model that the coefficients can be determined only from the tire physical parameters and its loading.

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